Model Predictive Control Toolbox™

Getting Started Guide

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R2021**a**

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Model Predictive Control Toolbox™ Getting Started Guide

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Revision History

October 2004	First printing	New for Version 2.1 (Release 14SP1)
March 2005	Online only	Revised for Version 2.2 (Release 14SP2)
September 2005	Online only	Revised for Version 2.2.1 (Release 14SP3)
March 2006	Online only	Revised for Version 2.2.2 (Release 2006a)
September 2006	Online only	Revised for Version 2.2.3 (Release 2006b)
March 2007	Online only	Revised for Version 2.2.4 (Release 2007a)
September 2007	Online only	Revised for Version 2.3 (Release 2007b)
March 2008	Online only	Revised for Version 2.3.1 (Release 2008a)
October 2008	Online only	Revised for Version 3.0 (Release 2008b)
March 2009	Online only	Revised for Version 3.1 (Release 2009a)
September 2009	Online only	Revised for Version 3.1.1 (Release 2009b)
March 2010	Online only	Revised for Version 3.2 (Release 2010a)
September 2010	Online only	Revised for Version 3.2.1 (Release 2010b)
April 2011	Online only	Revised for Version 3.3 (Release 2011a)
September 2011	Online only	Revised for Version 4.0 (Release 2011b)
March 2012	Online only	Revised for Version 4.1 (Release 2012a)
September 2012	Online only	Revised for Version 4.1.1 (Release 2012b)
March 2013	Online only	Revised for Version 4.1.2 (Release 2013a)
September 2013	Online only	Revised for Version 4.1.3 (Release 2013b)
March 2014	Online only	Revised for Version 4.2 (Release R2014a)
October 2014	Online only	Revised for Version 5.0 (Release 2014b)
March 2015	Online only	Revised for Version 5.0.1 (Release 2015a)
September 2015	Online only	Revised for Version 5.1 (Release 2015b)
March 2016	Online only	Revised for Version 5.2 (Release 2016a)
September 2016	Online only	Revised for Version 5.2.1 (Release 2016b)
March 2017	Online only	Revised for Version 5.2.2 (Release 2017a)
September 2017	Online only	Revised for Version 6.0 (Release 2017b)
March 2018	Online only	Revised for Version 6.1 (Release 2018a)
September 2018	Online only	Revised for Version 6.2 (Release 2018b)
March 2019	Online only	Revised for Version 6.3 (Release 2019a)
September 2019	Online only	Revised for Version 6.3.1 (Release 2019b)
March 2020	Online only	Revised for Version 6.4 (Release 2020a)
September 2020	Online only	Revised for Version 7.0 (Release 2020b)
March 2021	Online only	Revised for Version 7.1 (Release 2021a)

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Introduction

Model Predictive Control Toolbox Product Description

Design and simulate model predictive controllers

Model Predictive Control Toolbox provides functions, an app, and Simulink® blocks for designing and simulating controllers using linear and nonlinear model predictive control (MPC). The toolbox lets you specify plant and disturbance models, horizons, constraints, and weights. By running closed-loop simulations, you can evaluate controller performance.

You can adjust the behavior of the controller by varying its weights and constraints at run time. The toolbox provides deployable optimization solvers and also enables you to use a custom solver. To control a nonlinear plant, you can implement adaptive, gain-scheduled, and nonlinear MPC controllers. For applications with fast sample rates, the toolbox lets you generate an explicit model predictive controller from a regular controller or implement an approximate solution.

For rapid prototyping and embedded system implementation, including deployment of optimization solvers, the toolbox supports C code and IEC 61131-3 Structured Text generation.

Acknowledgments

MathWorks would like to acknowledge the following contributors to Model Predictive Control Toolbox.

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[4] Maciejowski, J. M., *Predictive Control with Constraints*, Pearson Education POD, 2002.

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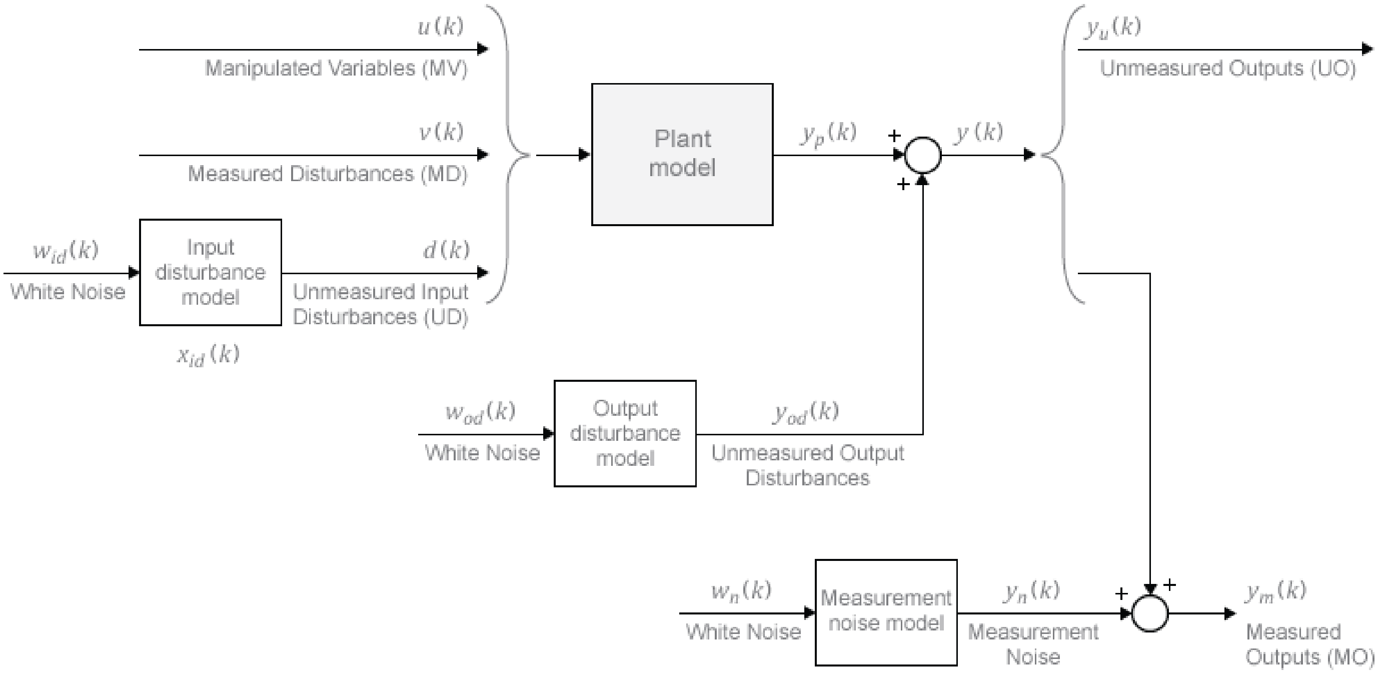
Building Models

MPC Modeling

Model predictive controllers use plant, disturbance, and noise models for prediction and state estimation. The model structure used in an MPC controller appears in the following illustration.



Plant Model

You can specify the plant model in one of the following linear-time-invariant (LTI) formats:

- Numeric LTI models — Transfer function (`tf`), state space (`ss`), zero-pole-gain (`zpk`)
- Identified models (requires System Identification Toolbox™) — `idss`, `idtf`, `idproc`, and `idpoly`

The MPC controller performs all estimation and optimization calculations using a discrete-time, delay-free, state-space system with dimensionless input and output variables. Therefore, when you specify a plant model in the MPC controller, the software performs the following, if needed:

1. Conversion to state space — The `ss` command converts the supplied model to an LTI state-space model.
2. Discretization or resampling — If the model sample time differs from the MPC controller sample time (defined in the `Ts` property), one of the following occurs:

   - If the model is continuous time, the `c2d` command converts it to a discrete-time LTI object using the controller sample time.
   - If the model is discrete time, the `d2d` command resamples it to generate a discrete-time LTI object using the controller sample time.
3. Delay removal — If the discrete-time model includes any input, output, or internal delays, the `absorbDelay` command replaces them with the appropriate number of poles at $z = 0$, increasing the total number of discrete states. The `InputDelay`, `OutputDelay`, and `InternalDelay` properties of the resulting state-space model are all zero.

4 Conversion to dimensionless input and output variables — The MPC controller enables you to specify a scale factor for each plant input and output variable. If you do not specify scale factors, they default to `1`. The software converts the plant input and output variables to dimensionless form as follows:

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B S_i u_p(k) \\ y_p(k) &= S_o^{-1} C x_p(k) + S_o^{-1} D S_i u_p(k)\,. \end{aligned}$$

where A_p, B, C, and D are the constant zero-delay state-space matrices from step 3, and:

- S_i is a diagonal matrix of input scale factors in engineering units.
- S_o is a diagonal matrix of output scale factors in engineering units.
- x_p is the state vector from step 3 in engineering units (including any absorbed delay states). No scaling is performed on state variables.
- u_p is a vector of dimensionless plant input variables, including manipulated variables, measured disturbances, and unmeasured input disturbances.
- y_p is a vector of dimensionless plant output variables.

The resulting plant model has the following equivalent form:

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B_{pu} u(k) + B_{pv} v(k) + B_{pd} d(k) \\ y_p(k) &= C_p x_p(k) + D_{pu} u(k) + D_{pv} v(k) + D_{pd} d(k)\,. \end{aligned}$$

Here, $C_p = S_o^{-1} C$, B_{pu}, B_{pv}, and B_{pd} are the corresponding columns of BS_i. Also, D_{pu}, D_{pv}, and D_{pd} are the corresponding columns of $S_o^{-1} D S_i$. Finally, $u(k)$, $v(k)$, and $d(k)$ are the dimensionless manipulated variables, measured disturbances, and unmeasured input disturbances, respectively.

The MPC controller enforces the restriction of $D_{pu} = 0$, which means that the controller does not allow direct feedthrough from any manipulated variable to any plant output.

Input Disturbance Model

If your plant model includes unmeasured input disturbances, $d(k)$, the input disturbance model specifies the signal type and characteristics of $d(k)$. See “Controller State Estimation” for more information about the model.

The `getindist` command provides access to the model in use.

The input disturbance model is a key factor that influences the following controller performance attributes:

- Dynamic response to apparent disturbances — The character of the controller response when the measured plant output deviates from its predicted trajectory, due to an unknown disturbance or modeling error.

- Asymptotic rejection of sustained disturbances — If the disturbance model predicts a sustained disturbance, controller adjustments continue until the plant output returns to its desired trajectory, emulating a classical integral feedback controller.

You can provide the input disturbance model as an LTI state-space (`ss`), transfer function (`tf`), or zero-pole-gain (`zpk`) object using `setindist`. The MPC controller converts the input disturbance model to a discrete-time, delay-free, LTI state-space system using the same steps used to convert the plant model on page 2-2. The result is:

$$\begin{aligned} x_{id}(k+1) &= A_{id}x_{id}(k) + B_{id}w_{id}(k) \\ d(k) &= C_{id}x_{id}(k) + D_{id}w_{id}(k)\,. \end{aligned}$$

where A_{id}, B_{id}, C_{id}, and D_{id} are constant state-space matrices, and:

- $x_{id}(k)$ is a vector of $n_{xid} \geq 0$ input disturbance model states.
- $d_k(k)$ is a vector of n_d dimensionless unmeasured input disturbances.
- $w_{id}(k)$ is a vector of $n_{id} \geq 1$ dimensionless white noise inputs, assumed to have zero mean and unit variance.

If you do not provide an input disturbance model, then the controller uses a default model, which has integrators with dimensionless unity gain added to its outputs. An integrator is added for each unmeasured input disturbance, unless doing so would cause a violation of state observability. In this case, a static system with dimensionless unity gain is used instead.

Output Disturbance Model

The output disturbance model is a special case of the more general input disturbance model. Its output, $y_{od}(k)$, is directly added to the plant output rather than affecting the plant states. The output disturbance model specifies the signal type and characteristics of $y_{od}(k)$, and it is often used in practice. See “Controller State Estimation” for more details about the model.

The `getoutdist` command provides access to the output disturbance model in use.

You can specify a custom output disturbance model as an LTI state-space (`ss`), transfer function (`tf`), or zero-pole-gain (`zpk`) object using `setoutdist`. Using the same steps as for the plant model on page 2-2, the MPC controller converts the specified output disturbance model to a discrete-time, delay-free, LTI state-space system. The result is:

$$\begin{aligned} x_{od}(k+1) &= A_{od}x_{od}(k) + B_{od}w_{od}(k) \\ y_{od}(k) &= C_{od}x_{od}(k) + D_{od}w_{od}(k)\,. \end{aligned}$$

where A_{od}, B_{od}, C_{od}, and D_{od} are constant state-space matrices, and:

- $x_{od}(k)$ is a vector of $n_{xod} \geq 1$ output disturbance model states.
- $y_{od}(k)$ is a vector of n_y dimensionless output disturbances to be added to the dimensionless plant outputs.
- $w_{od}(k)$ is a vector of n_{od} dimensionless white noise inputs, assumed to have zero mean and unit variance.

If you do not specify an output disturbance model, then the controller uses a default model, which has integrators with dimensionless unity gain added to some or all of its outputs. These integrators are added according to the following rules:

- No disturbances are estimated, that is no integrators are added, for unmeasured plant outputs.
- An integrator is added for each measured output in order of decreasing output weight.
  - For time-varying weights, the sum of the absolute values over time is considered for each output channel.

  - For equal output weights, the order within the output vector is followed.
- For each measured output, an integrator is not added if doing so would cause a violation of state observability. Instead, a gain with a value of zero is used instead.

If there is an input disturbance model, then the controller adds any default integrators to that model before constructing the default output disturbance model.

Measurement Noise Model

One controller design objective is to distinguish disturbances, which require a response, from measurement noise, which should be ignored. The measurement noise model specifies the expected noise type and characteristics. See “Controller State Estimation” for more details about the model.

Using the same steps as for the plant model on page 2-2, the MPC controller converts the measurement noise model to a discrete-time, delay-free, LTI state-space system. The result is:

$$\begin{aligned} x_n(k+1) &= A_n x_n(k) + B_n w_n(k) \\ y_n(k) &= C_n x_n(k) + D_n w_n(k)\,. \end{aligned}$$

Here, A_n, B_n, C_n, and D_n are constant state space matrices, and:

- $x_n(k)$ is a vector of $n_{xn} \geq 0$ noise model states.
- $y_n(k)$ is a vector of n_{ym} dimensionless noise signals to be added to the dimensionless measured plant outputs.
- $w_n(k)$ is a vector of $n_n \geq 1$ dimensionless white noise inputs, assumed to have zero mean and unit variance.

If you do not supply a noise model, the default is a unity static gain: $n_{xn} = 0$, D_n is an n_{ym}-by-n_{ym} identity matrix, and A_n, B_n, and C_n are empty.

For an `mpc` controller object, `MPCobj`, the property `MPCobj.Model.Noise` provides access to the measurement noise model.

Note If the minimum eigenvalue of $D_n D_n^T$ is less than 1x10^{-8}, the MPC controller adds 1x10^{-4} to each diagonal element of D_n. This adjustment makes a successful default Kalman gain calculation more likely.

See Also

More About

- “Controller State Estimation”
- “Adjust Disturbance and Noise Models”

Signal Types

Inputs

The *plant inputs* are the independent variables affecting the plant. As shown in “MPC Modeling” on page 2-2, there are three types:

Measured disturbances

The controller can't adjust them, but uses them for feedforward compensation.

Manipulated variables

The controller adjusts these in order to achieve its goals.

Unmeasured disturbances

These are independent inputs of which the controller has no direct knowledge, and for which it must compensate.

Outputs

The *plant outputs* are the dependent variables (outcomes) you wish to control or monitor. As shown in “MPC Modeling” on page 2-2, there are two types:

Measured outputs

The controller uses these to estimate unmeasured quantities and as feedback on the success of its adjustments.

Unmeasured outputs

The controller estimates these based on available measurements and the plant model. The controller can also hold unmeasured outputs at setpoints or within constraint boundaries.

You must specify the input and output types when designing the controller. See “Input and Output Types” on page 2-10 for more details.

See Also

More About

- “MPC Modeling” on page 2-2

Construct Linear Time Invariant Models

Model Predictive Control Toolbox software supports the same LTI model formats as does Control System Toolbox software. You can use whichever is most convenient for your application and convert from one format to another. For more details, see “Basic Models”.

Transfer Function Models

A transfer function (TF) relates a particular input/output pair. For example, if $u(t)$ is a plant input and $y(t)$ is an output, the transfer function relating them might be:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{s+2}{s^2+s+10}e^{-1.5s}$$

This TF consists of a *numerator* polynomial, s+2, a *denominator* polynomial, s^2+s+10, and a delay, which is 1.5 time units here. You can define G using Control System Toolbox `tf` function:

```
Gtf1 = tf([1 2], [1 1 10],'OutputDelay',1.5)
```

```
Transfer function:
                  s + 2
exp(-1.5*s) * ------------
               s^2 + s + 10
```

Zero/Pole/Gain Models

Like the TF format, the zero/pole/gain (ZPK) format relates an input/output pair. The difference is that the ZPK numerator and denominator polynomials are factored, as in

$$G(s) = 2.5\frac{s+0.45}{(s+0.3)(s+0.1+0.7i)(s+0.1-0.7i)}$$

(zeros and/or poles are complex numbers in general).

You define the ZPK model by specifying the zero(s), pole(s), and gain as in

```
poles = [-0.3, -0.1+0.7*i, -0.1-0.7*i];
Gzpk1 = zpk(-0.45,poles,2.5);
```

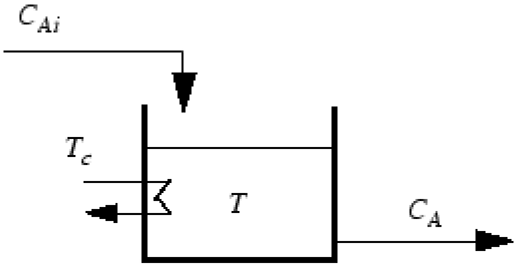
State-Space Models

The state-space format is convenient if your model is a set of LTI differential and algebraic equations. For example, consider the following linearized model of a continuous stirred-tank reactor (CSTR) involving an exothermic (heat-generating) reaction [1].

$$\frac{dC'_A}{dt} = a_{11}C'_A + a_{12}T' + b_{11}T'_c + b_{12}C'_{Ai}$$

$$\frac{dT'}{dt} = a_{21}C'_A + a_{22}T' + b_{21}T'_c + b_{22}C'_{Ai}$$

where C_A is the concentration of a key reactant, T is the temperature in the reactor, T_c is the coolant temperature, C_{Ai} is the reactant concentration in the reactor feed, and a_{ij} and b_{ij} are constants. See the process schematic in "CSTR Schematic" on page 2-8. The primes (e.g., C'_A) denote a deviation from the nominal steady-state condition at which the model has been linearized.



CSTR Schematic

Measurement of reactant concentrations is often difficult, if not impossible. Let us assume that T is a measured output, C_A is an unmeasured output, T_c is a manipulated variable, and C_{Ai} is an unmeasured disturbance.

The model fits the general state-space format

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where

$$x = \begin{bmatrix} C'_A \\ T' \end{bmatrix}, u = \begin{bmatrix} T'_c \\ C'_{Ai} \end{bmatrix}, y = \begin{bmatrix} T' \\ C'_A \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The following code shows how to define such a model for some specific values of the a_{ij} and b_{ij} constants:

```
A = [-0.0285  -0.0014
     -0.0371  -0.1476];
B = [-0.0850   0.0238
      0.0802   0.4462];
C = [0 1
     1 0];
D = zeros(2,2);
CSTR = ss(A,B,C,D);
```

This defines a *continuous-time* state-space model. If you do not specify a sampling period, a default sampling value of zero applies. You can also specify discrete-time state-space models. You can specify delays in both continuous-time and discrete-time models.

Note In the CSTR example, the D matrix is zero and the output does not instantly respond to change in the input. The Model Predictive Control Toolbox software prohibits direct (instantaneous) feedthrough from a manipulated variable to an output. For example, the CSTR model could include direct feedthrough from the unmeasured disturbance, C_{Ai}, to either C_A or T but direct feedthrough from T_c to either output would violate this restriction. If the model had direct feedthrough from T_c, you can add a small delay at this input to circumvent the problem.

LTI Object Properties

The `ss` function in the last line of the above code creates a state-space model, `CSTR`, which is an *LTI object*. The `tf` and `zpk` commands described in “Transfer Function Models” on page 2-7 and “Zero/Pole/Gain Models” on page 2-7 also create LTI objects. Such objects contain the model parameters as well as optional properties.

LTI Properties for the CSTR Example

The following code sets some of the `CSTR` model's optional properties:

```
CSTR.InputName = {'T_c','C_A_i'};
CSTR.OutputName = {'T','C_A'};
CSTR.StateName = {'C_A','T'};
CSTR.InputGroup.MV = 1;
CSTR.InputGroup.UD = 2;
CSTR.OutputGroup.MO = 1;
CSTR.OutputGroup.UO = 2;
CSTR
```

The first three lines specify labels for the input, output and state variables. The next four specify the signal type for each input and output. The designations `MV`, `UD`, `MO`, and `UO` mean *manipulated variable*, *unmeasured disturbance*, *measured output*, and *unmeasured output*. (See “Signal Types” on page 2-6 for definitions.) For example, the code specifies that input 2 of model `CSTR` is an unmeasured disturbance. The last line causes the LTI object to be displayed, generating the following lines in the MATLAB® Command Window:

```
A =
            C_A        T
   C_A  -0.0285  -0.0014
   T    -0.0371  -0.1476

B =
           T_c    C_Ai
   C_A  -0.085  0.0238
   T    0.0802  0.4462

C =
        C_A    T
   T      0    1
   C_A    1    0


D =
         T_c  C_Ai
   T       0     0
   C_A     0     0

Input groups:
    Name    Channels
     MV        1
     UD        2
```

```
Output groups:
    Name    Channels
     MO        1
     UO        2

Continuous-time model
```

Input and Output Names

The optional `InputName` and `OutputName` properties affect the model displays, as in the above example. The software also uses the `InputName` and `OutputName` properties to label plots and tables. In that context, the underscore character causes the next character to be displayed as a subscript.

Input and Output Types

General Case

As mentioned in “Signal Types” on page 2-6, Model Predictive Control Toolbox software supports three input types and two output types. In a Model Predictive Control Toolbox design, designation of the input and output types determines the controller dimensions and has other important consequences.

For example, suppose your plant structure were as follows:

Plant Inputs	Plant Outputs
Two manipulated variables (MVs)	Three measured outputs (MOs)
One measured disturbance (MD)	Two unmeasured outputs (UOs)
Two unmeasured disturbances (UDs)	

The resulting controller has four inputs (the three MOs and the MD) and two outputs (the MVs). It includes feedforward compensation for the measured disturbance, and assumes that you wanted to include the unmeasured disturbances and outputs as part of the regulator design.

If you didn't want a particular signal to be treated as one of the above types, you could do one of the following:

- Eliminate the signal before using the model in controller design.
- For an output, designate it as unmeasured, then set its weight to zero.
- For an input, designate it as an unmeasured disturbance, then define a custom state estimator that ignores the input.

Note By default, the software assumes that unspecified plant inputs are manipulated variables, and unspecified outputs are measured. Thus, if you didn't specify signal types in the above example, the controller would have four inputs (assuming all plant outputs were measured) and five outputs (assuming all plant inputs were manipulated variables).

For model `CSTR`, the default Model Predictive Control Toolbox assumptions are incorrect. You must set its `InputGroup` and `OutputGroup` properties, as illustrated in the above code, or modify the default settings when you load the model into **MPC Designer**.

Use `setmpcsignals` to make type definition. For example:

```
CSTR = setmpcsignals(CSTR,'UD',2,'UO',2);
```

sets InputGroup and OutputGroup to the same values as in the previous example. The CSTR display would then include the following lines:

```
Input groups:
       Name        Channels
    Unmeasured        2
    Manipulated       1



Output groups:
       Name       Channels
    Unmeasured       2
     Measured        1
```

Notice that setmpcsignals sets unspecified inputs to Manipulated and unspecified outputs to Measured.

LTI Model Characteristics

Control System Toolbox software provides functions for analyzing LTI models. Some of the more commonly used are listed below. Type the example code at the MATLAB prompt to see how they work for the CSTR example.

Example	Intended Result
dcgain(CSTR)	Calculate gain matrix for the CSTR model's input/output pairs.
impulse(CSTR)	Graph CSTR model's unit-impulse response.
linearSystemAnalyzer(CSTR)	Open the Linear System Analyzer with the CSTR model loaded. You can then display model characteristics by making menu selections.
pole(CSTR)	Calculate CSTR model's poles (to check stability, etc.).
step(CSTR)	Graph CSTR model's unit-step response.
zero(CSTR)	Compute CSTR model's transmission zeros.

References

[1] Seborg, D. E., T. F. Edgar, and D. A. Mellichamp, *Process Dynamics and Control*, 2nd Edition, Wiley, 2004, pp. 34–36 and 94–95.

See Also

setmpcsignals | ss | tf | zpk

More About

- “Specify Multi-Input Multi-Output Plants” on page 2-12

Specify Multi-Input Multi-Output Plants

Most MPC applications involve plants with multiple inputs and outputs. You can use `ss`, `tf`, and `zpk` to represent a MIMO plant model. For example, consider the following model of a distillation column [1], which has been used in many advanced control studies:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21.0s+1} & \dfrac{3.8e^{-8.1s}}{14.9s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} & \dfrac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Outputs y_1 and y_2 represent measured product purities. The controller manipulates the inputs, u_1 and u_2, to hold each output at a specified setpoint. These inputs represent the flow rates of reflux and reboiler steam, respectively. Input u_3 is a measured feed flow rate disturbance.

The model consists of six transfer functions, one for each input/output pair. Each transfer function is the first-order-plus-delay form often used by process control engineers.

Specify the individual transfer functions for each input/output pair. For example, `g12` is the transfer function from input u_1 to output y_2.

```
g11 = tf( 12.8, [16.7 1], 'IOdelay', 1.0,'TimeUnit','minutes');
g12 = tf(-18.9, [21.0 1], 'IOdelay', 3.0,'TimeUnit','minutes');
g13 = tf(  3.8, [14.9 1], 'IOdelay', 8.1,'TimeUnit','minutes');
g21 = tf(  6.6, [10.9 1], 'IOdelay', 7.0,'TimeUnit','minutes');
g22 = tf(-19.4, [14.4 1], 'IOdelay', 3.0,'TimeUnit','minutes');
g23 = tf(  4.9, [13.2 1], 'IOdelay', 3.4,'TimeUnit','minutes');
```

Define a MIMO system by creating a matrix of transfer function models.

```
DC = [g11 g12 g13
      g21 g22 g23];
```

Define the input and output signal names and specify the third input as a measured input disturbance.

```
DC.InputName = {'Reflux Rate','Steam Rate','Feed Rate'};
DC.OutputName = {'Distillate Purity','Bottoms Purity'};
DC = setmpcsignals(DC,'MD',3);
```

```
-->Assuming unspecified input signals are manipulated variables.
```

Review the resulting system.

```
DC
```

```
DC =

  From input "Reflux Rate" to output...
                                      12.8
   Distillate Purity:  exp(-1*s) * ----------
                                   16.7 s + 1

                                   6.6
   Bottoms Purity:  exp(-7*s) * ----------
                                10.9 s + 1
  From input "Steam Rate" to output...
                                    -18.9
   Distillate Purity:  exp(-3*s) * --------
                                   21 s + 1

                                  -19.4
   Bottoms Purity:  exp(-3*s) * ----------
                                14.4 s + 1

  From input "Feed Rate" to output...
                                        3.8
   Distillate Purity:  exp(-8.1*s) * ----------
                                     14.9 s + 1

                                     4.9
   Bottoms Purity:  exp(-3.4*s) * ----------
                                  13.2 s + 1

Input groups:
       Name        Channels
     Measured         3
    Manipulated      1,2

Output groups:
      Name      Channels
    Measured      1,2

Continuous-time transfer function.
```

References

[1] Wood, R. K., and M. W. Berry, *Chem. Eng. Sci.*, Vol. 28, pp. 1707, 1973.

See Also

`setmpcsignals` | `ss` | `tf` | `zpk`

Related Examples

- “Construct Linear Time Invariant Models” on page 2-7

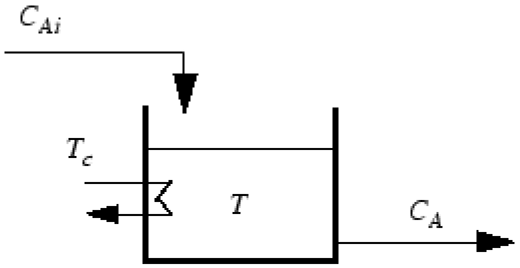
CSTR Model

The linearized model of a continuous stirred-tank reactor (CSTR) involving an exothermic (heat-generating) reaction is represented by the following differential equations:

$$\frac{dC'_A}{dt} = a_{11}C'_A + a_{12}T' + b_{11}T'_c + b_{12}C'_{Ai}$$

$$\frac{dT'}{dt} = a_{21}C'_A + a_{22}T' + b_{21}T'_c + b_{22}C'_{Ai}$$

where C_A is the concentration of a key reactant, T is the temperature in the reactor, T_c is the coolant temperature, C_{Ai} is the reactant concentration in the reactor feed, and a_{ij} and b_{ij} are constants. The primes (e.g., C'_A) denote a deviation from the nominal steady-state condition at which the model has been linearized.



Measurement of reactant concentrations is often difficult, if not impossible. Let us assume that T is a measured output, C_A is an unmeasured output, T_c is a manipulated variable, and C_{Ai} is an unmeasured disturbance.

The model fits the general state-space format

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where

$$x = \begin{bmatrix} C'_A \\ T' \end{bmatrix}, u = \begin{bmatrix} T'_c \\ C'_{Ai} \end{bmatrix}, y = \begin{bmatrix} T' \\ C'_A \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The following code shows how to define such a model for some specific values of the a_{ij} and b_{ij} constants:

```
A = [-0.0285  -0.0014
     -0.0371  -0.1476];
B = [-0.0850   0.0238
      0.0802   0.4462];
C = [0 1
     1 0];
```

```
D = zeros(2,2);
CSTR = ss(A,B,C,D);
```

The following code sets some of the CSTR model's optional properties:

```
CSTR.InputName = {'T_c', 'C_A_i'};
CSTR.OutputName = {'T', 'C_A'};
CSTR.StateName = {'C_A', 'T'};
CSTR.InputGroup.MV = 1;
CSTR.InputGroup.UD = 2;
CSTR.OutputGroup.MO = 1;
CSTR.OutputGroup.UO = 2;
```

To view the properties of CSTR, enter:

```
CSTR
```

Linearize Simulink Models

Generally, real systems are nonlinear. To design an MPC controller for a nonlinear system, you can model the plant in Simulink.

Although an MPC controller can regulate a nonlinear plant, the model used within the controller must be linear. In other words, the controller employs a linear approximation of the nonlinear plant. The accuracy of this approximation significantly affects controller performance.

To obtain such a linear approximation, you *linearize* the nonlinear plant at a specified *operating point*.

Note Simulink Control Design software must be installed to linearize nonlinear Simulink models.

You can linearize a Simulink model:

- From the command line.
- Using the **Model Linearizer**.
- Using **MPC Designer**. For an example, see "Linearize Simulink Models Using MPC Designer" on page 2-23.

Linearization Using MATLAB Code

This example shows how to obtain a linear model of a plant using a MATLAB script.

For this example the CSTR model, `CSTR_OpenLoop`, is linearized. The model inputs are the coolant temperature (manipulated variable of the MPC controller), limiting reactant concentration in the feed stream, and feed temperature. The model states are the temperature and concentration of the limiting reactant in the product stream. Both states are measured and used for feedback control.

Obtain Steady-State Operating Point

The operating point defines the nominal conditions at which you linearize a model. It is usually a steady-state condition.

Suppose that you plan to operate the CSTR with the output concentration, `C_A`, at 2 $kmol/m^3$. The nominal feed concentration is 10 $kmol/m^3$, and the nominal feed temperature is 300 K. Create an operating point specification object to define the steady-state conditions.

```
opspec = operspec('CSTR_OpenLoop');
opspec = addoutputspec(opspec,'CSTR_OpenLoop/CSTR',2);
opspec.Outputs(1).Known = true;
opspec.Outputs(1).y = 2;

op1 = findop('CSTR_OpenLoop',opspec);
```

```
 Operating point search report:
---------------------------------

 Operating point search report for the Model CSTR_OpenLoop.
 (Time-Varying Components Evaluated at time t=0)

Operating point specifications were successfully met.
```

```
States:
----------
(1.) CSTR_OpenLoop/CSTR/C_A
      x:             2      dx:      -4.6e-12 (0)
(2.) CSTR_OpenLoop/CSTR/T_K
      x:           373      dx:      5.49e-11 (0)

Inputs:
----------
(1.) CSTR_OpenLoop/Coolant Temperature
      u:           299    [-Inf Inf]

Outputs:
----------
(1.) CSTR_OpenLoop/CSTR
      y:             2    (2)
```

The calculated operating point is `C_A` = 2 $kmol/m^3$ and `T_K` = 373 K. Notice that the steady-state coolant temperature is also given as 299 K, which is the nominal value of the manipulated variable of the MPC controller.

To specify:

- Values of known inputs, use the `Input.Known` and `Input.u` fields of `opspec`
- Initial guesses for state values, use the `State.x` field of `opspec`

For example, the following code specifies the coolant temperature as 305 K and initial guess values of the `C_A` and `T_K` states before calculating the steady-state operating point:

```
opspec = operspec('CSTR_OpenLoop');
opspec.States(1).x = 1;
opspec.States(2).x = 400;
opspec.Inputs(1).Known = true;
opspec.Inputs(1).u = 305;

op2 = findop('CSTR_OpenLoop',opspec);
```

```
 Operating point search report:
---------------------------------

 Operating point search report for the Model CSTR_OpenLoop.
 (Time-Varying Components Evaluated at time t=0)

Operating point specifications were successfully met.
States:
----------
(1.) CSTR_OpenLoop/CSTR/C_A
      x:          1.78      dx:     -1.42e-14 (0)
(2.) CSTR_OpenLoop/CSTR/T_K
      x:           377      dx:      5.68e-14 (0)

Inputs:
----------
(1.) CSTR_OpenLoop/Coolant Temperature
      u:           305

Outputs: None
----------
```

Specify Linearization Inputs and Outputs

If the linearization input and output signals are already defined in the model, as in CSTR_OpenLoop, then use the following to obtain the signal set.

```
io = getlinio('CSTR_OpenLoop');
```

Otherwise, specify the input and output signals as shown here.

```
io(1) = linio('CSTR_OpenLoop/Coolant Temperature',1,'input');
io(2) = linio('CSTR_OpenLoop/Feed Concentration',1,'input');
io(3) = linio('CSTR_OpenLoop/Feed Temperature',1,'input');
io(4) = linio('CSTR_OpenLoop/CSTR',1,'output');
io(5) = linio('CSTR_OpenLoop/CSTR',2,'output');
```

Linearize Model

Linearize the model using the specified operating point, op1, and input/output signals, io.

```
sys = linearize('CSTR_OpenLoop',op1,io)
```

```
sys =

  A =
            C_A      T_K
   C_A       -5  -0.3427
   T_K    47.68    2.785

  B =
        Coolant Temp  Feed Concent  Feed Tempera
   C_A             0             1             0
   T_K           0.3             0             1

  C =
          C_A  T_K
   CSTR/1    0    1
   CSTR/2    1    0

  D =
          Coolant Temp  Feed Concent  Feed Tempera
   CSTR/1             0             0             0
   CSTR/2             0             0             0

Continuous-time state-space model.
```

Linearization Using Model Linearizer in Simulink Control Design

This example shows how to linearize a Simulink model using the **Model Linearizer**, provided by the Simulink Control Design software.

Open Simulink Model

This example uses the CSTR model, CSTR_OpenLoop.

```
open_system('CSTR_OpenLoop')
```

Specify Linearization Inputs and Outputs

The linearization inputs and outputs are already specified for `CSTR_OpenLoop`. The input signals correspond to the outputs from the `Feed Concentration`, `Feed Temperature`, and `Coolant Temperature` blocks. The output signals are the inputs to the `CSTR Temperature` and `Residual Concentration` blocks.

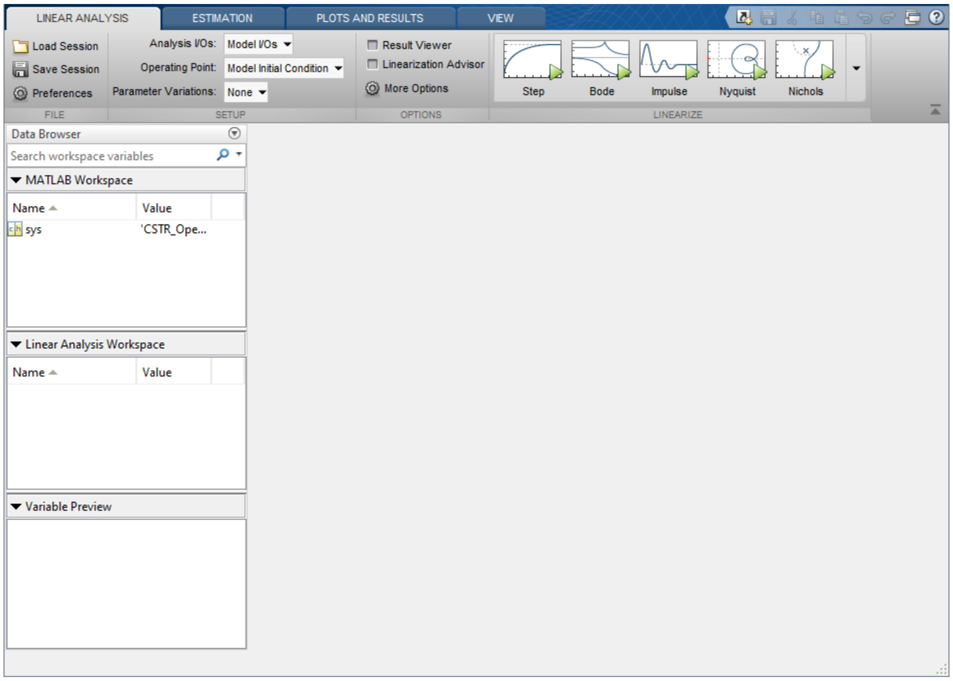
To specify a signal as a linearization input or output, first open the **Linearization** tab. To do so, in the Simulink **Apps** gallery, click **Linearization Manager**. Then, in the Simulink model window, click the signal.

To specify the signal as a:

- Linearization input, on the **Linearization** tab, in the **Insert Analysis Points** gallery, click **Input Perturbation**.
- Linearization output, on the **Linearization** tab, in the **Insert Analysis Points** gallery, click **Output Measurement**.

Open Model Linearizer

To open the **Model Linearizer**, in the **Apps** gallery, click **Model Linearizer**.

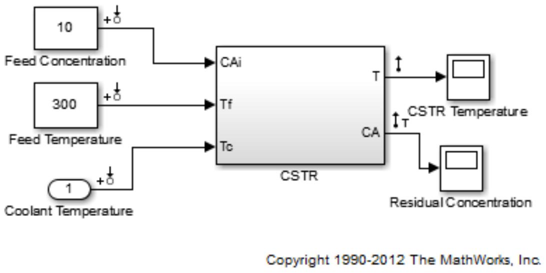


Analysis I/Os: Model I/Os
Operating Point: Model Initial Condition
Parameter Variations: None

Name	Value
sys	'CSTR_Ope...

Specify Residual Concentration as Known Trim Constraint

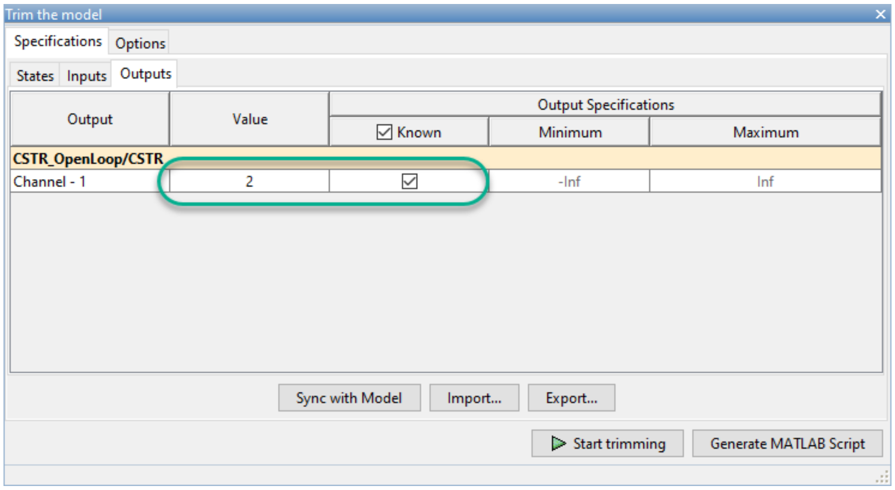
In the Simulink model window, click the `CA` output signal from the `CSTR` block. Then, on the **Linearization** tab, in the **Insert Analysis Points** gallery, click **Trim Output Constraint**.



In the **Model Linearizer**, on the **Linear Analysis** tab, select **Operating Point** > **Trim Model**.

In the Trim the model dialog box, on the **Outputs** tab:

- Select the **Known** check box for `Channel - 1` under **CSTR_OpenLoop/CSTR**.
- Set the corresponding **Value** to 2 kmol/m^3.

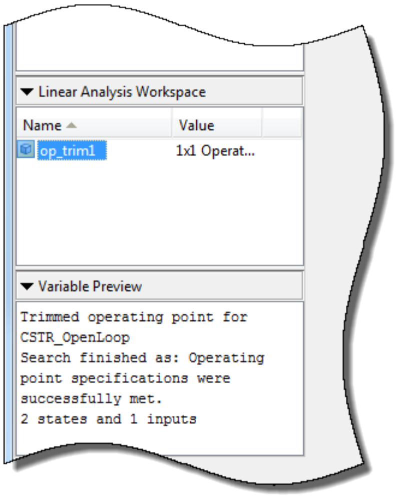


Output	Value	Output Specifications		
		☑ Known	Minimum	Maximum
CSTR_OpenLoop/CSTR				
Channel - 1	2	☑	-Inf	Inf

Create and Verify Operating Point

In the Trim the model dialog box, click **Start trimming**.

The operating point `op_trim1` displays in the **Linear Analysis Workspace**.

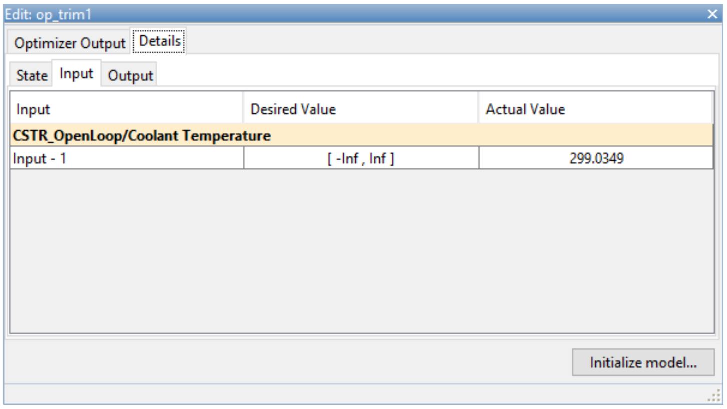


Name	Value
op_trim1	1x1 Operat...

```
Trimmed operating point for
CSTR_OpenLoop
Search finished as: Operating
point specifications were
successfully met.
2 states and 1 inputs
```

Double click `op_trim1` to view the resulting operating point.

In the Edit dialog box, select the **Input** tab.



Input	Desired Value	Actual Value
CSTR_OpenLoop/Coolant Temperature		
Input - 1	[-Inf , Inf]	299.0349

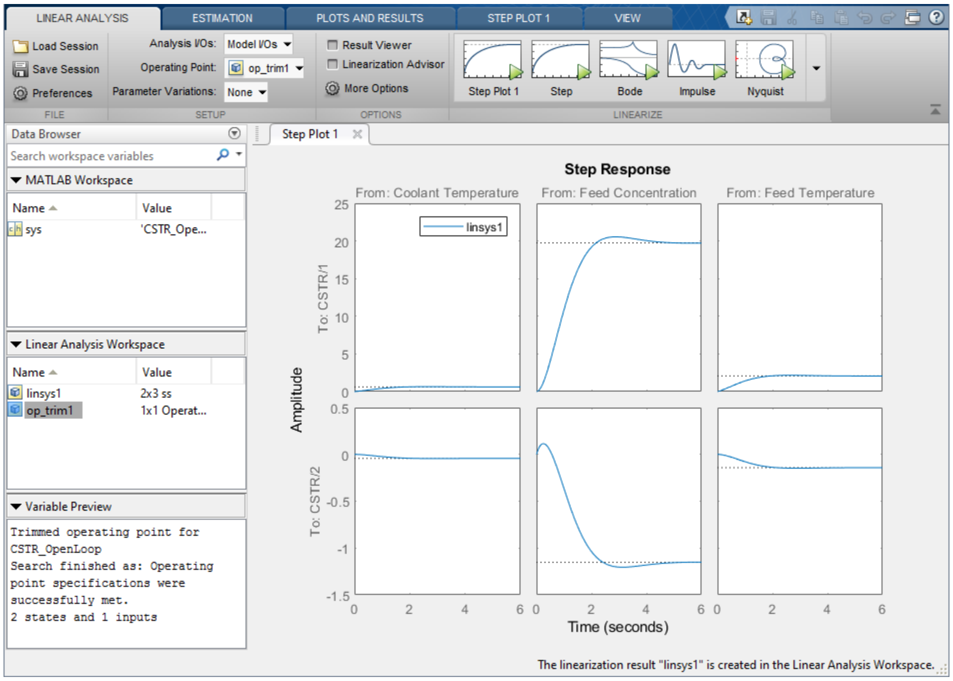
The coolant temperature at steady state is 299 K, as desired.

Linearize Model

On the **Linear Analysis** tab, in the **Operating Point** drop-down list, select `op_trim1`.

Click **Step** to linearize the model.

This option creates the linear model `linsys1` in the **Linear Analysis Workspace** and generates a step response for this model. `linsys1` uses `optrim1` as its operating point.



Analysis I/Os: Model I/Os
Operating Point: op_trim1
Parameter Variations: None

MATLAB Workspace

Name	Value
sys	'CSTR_Ope...

Linear Analysis Workspace

Name	Value
linsys1	2x3 ss
op_trim1	1x1 Operat...

```
Trimmed operating point for
CSTR_OpenLoop
Search finished as: Operating
point specifications were
successfully met.
2 states and 1 inputs
```

The linearization result "linsys1" is created in the Linear Analysis Workspace.

The step response from feed concentration to output CSTR/2 displays an interesting inverse response. An examination of the linear model shows that CSTR/2 is the residual CSTR concentration, C_A. When the feed concentration increases, C_A increases initially because more reactant is entering, which increases the reaction rate. This rate increase results in a higher reactor temperature (output CSTR/1), which further increases the reaction rate and C_A decreases dramatically.

Export Linearization Result

If necessary, you can repeat any of these steps to improve your model performance. Once you are satisfied with your linearization result, in the **Model Linearizer**, drag and drop it from the **Linear Analysis Workspace** to the **MATLAB Workspace**. You can now use your linear model to design an MPC controller.

See Also

Model Linearizer | `linearize`

Related Examples

- “Design MPC Controller in Simulink” on page 3-31
- “Design Controller Using MPC Designer” on page 3-2
- “Design MPC Controller at the Command Line” on page 3-20

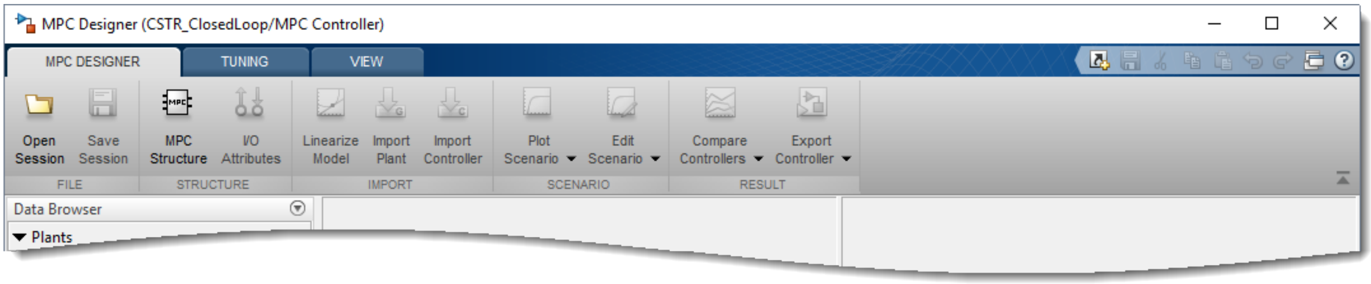
Linearize Simulink Models Using MPC Designer

This topic shows how to linearize Simulink models using **MPC Designer**. To do so, open the app from a Simulink model that contains an MPC Controller block. For this example, use the `CSTR_ClosedLoop` model.

```
sys = 'CSTR_ClosedLoop';
open_system(sys)
```

In the model window, double-click the MPC Controller block.

In the Block Parameters dialog box, ensure that the **MPC Controller** field is empty, and click **Design** to open **MPC Designer**.



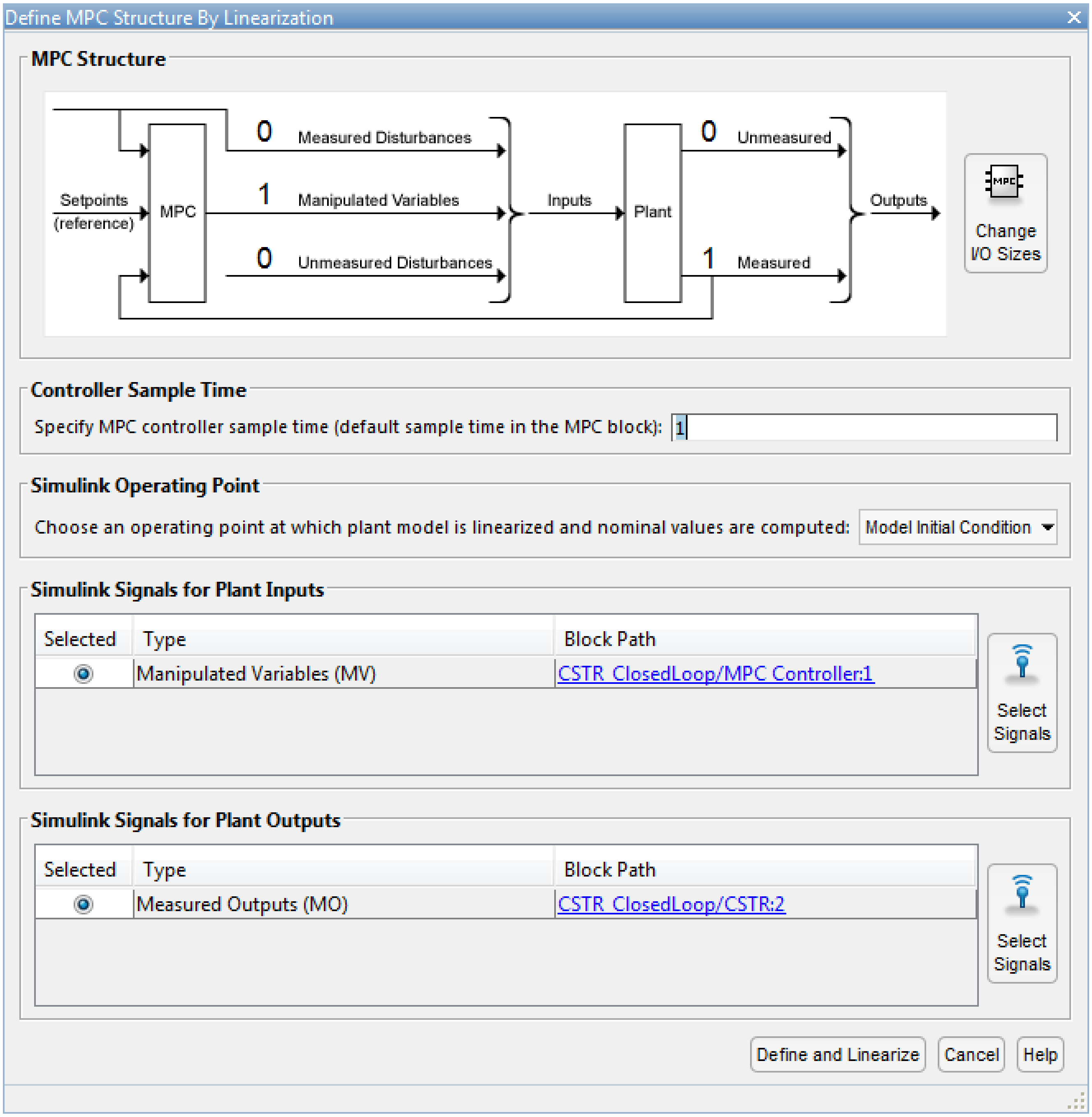
Using **MPC Designer**, you can define the MPC structure by linearizing the Simulink model. After you define the initial MPC structure, you can also linearize the model at different operating points and import the linearized plants.

Note If a controller from the MATLAB workspace is specified in the **MPC Controller** field, the app imports the specified controller. In this case, the MPC structure is derived from the imported controller. In this case, you can still linearize the Simulink model and import the linearized plants.

Define MPC Structure By Linearization

This example shows how to define the plant input/output structure in **MPC Designer** by linearizing a Simulink model.

On the **MPC Designer** tab, in the **Structure** section, click **MPC Structure**.



Define MPC Structure By Linearization

MPC Structure

Controller Sample Time

Specify MPC controller sample time (default sample time in the MPC block): 1

Simulink Operating Point

Choose an operating point at which plant model is linearized and nominal values are computed: Model Initial Condition

Simulink Signals for Plant Inputs

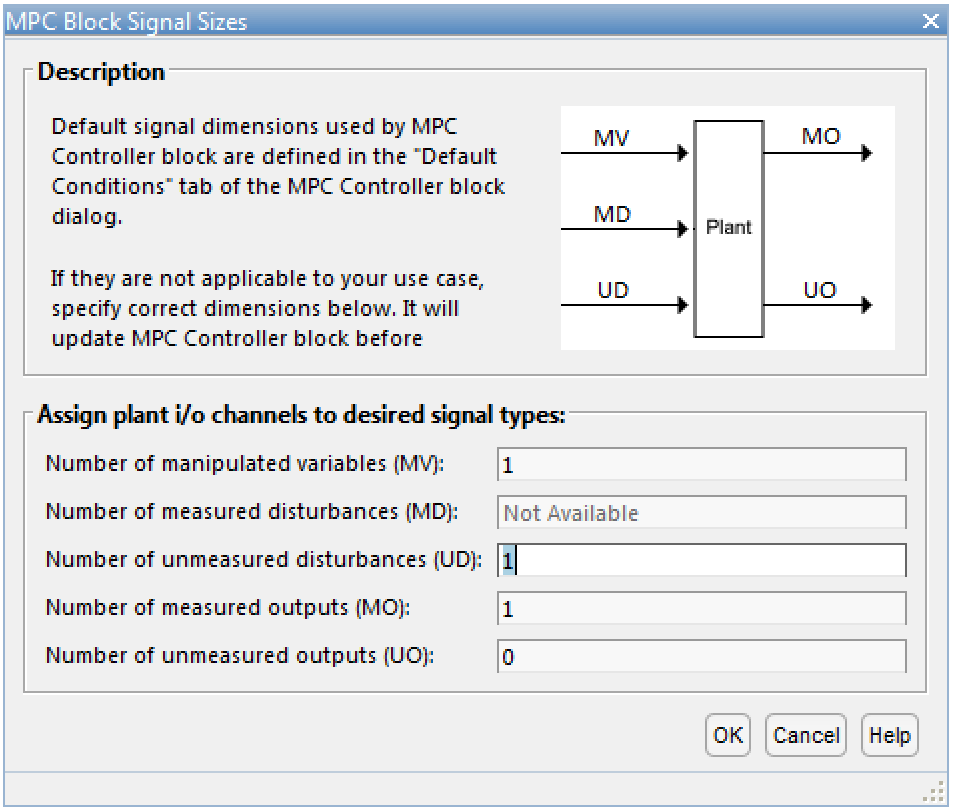
Selected	Type	Block Path
●	Manipulated Variables (MV)	CSTR_ClosedLoop/MPC Controller:1

Simulink Signals for Plant Outputs

Selected	Type	Block Path
●	Measured Outputs (MO)	CSTR_ClosedLoop/CSTR:2

Specify Signal Dimensions

In the Define MPC Structure By Linearization dialog box, in the **MPC Structure** section, if the displayed signal dimensions do not match your model, click **Change I/O Sizes** to configure the dimensions. Any unmeasured disturbances or unmeasured outputs in your model are not detected by the MPC Controller block. Specify the dimensions for these signals.



MPC Block Signal Sizes

Description

Default signal dimensions used by MPC Controller block are defined in the "Default Conditions" tab of the MPC Controller block dialog.

If they are not applicable to your use case, specify correct dimensions below. It will update MPC Controller block before

Assign plant i/o channels to desired signal types:

Number of manipulated variables (MV): 1

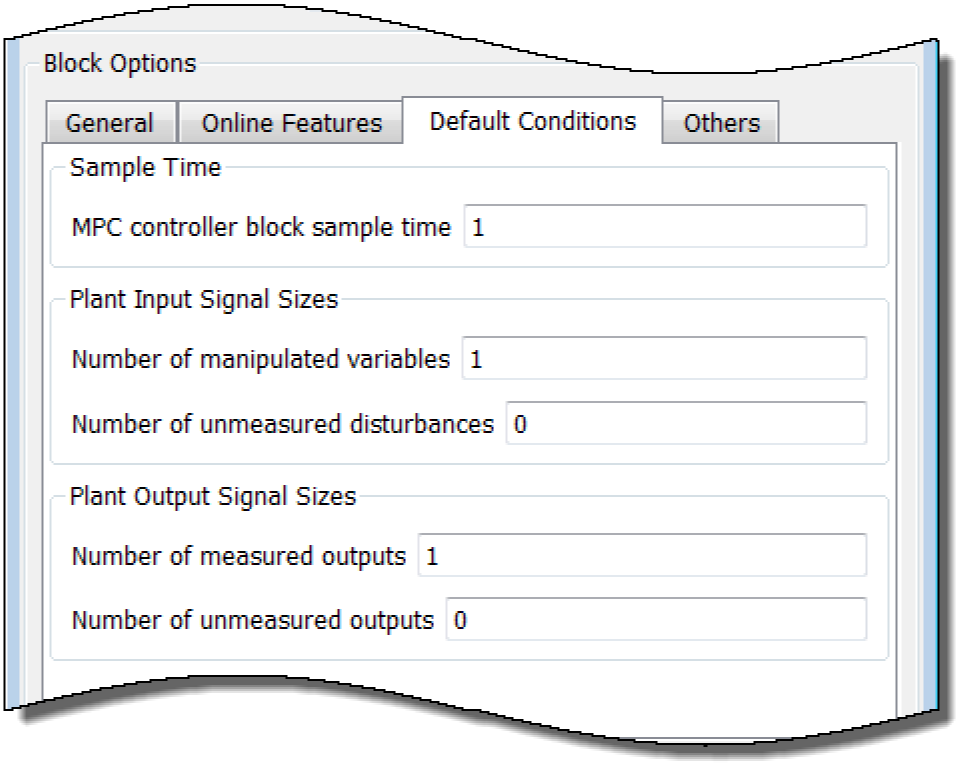
Number of measured disturbances (MD): Not Available

Number of unmeasured disturbances (UD): 1

Number of measured outputs (MO): 1

Number of unmeasured outputs (UO): 0

Tip In the MPC Controller Block Parameters dialog box, in the **Default Conditions** tab, you can define the controller sample time and signal dimensions before opening **MPC Designer**.



Sample Time

MPC controller block sample time: 1

Plant Input Signal Sizes

Number of manipulated variables: 1

Number of unmeasured disturbances: 0

Plant Output Signal Sizes

Number of measured outputs: 1

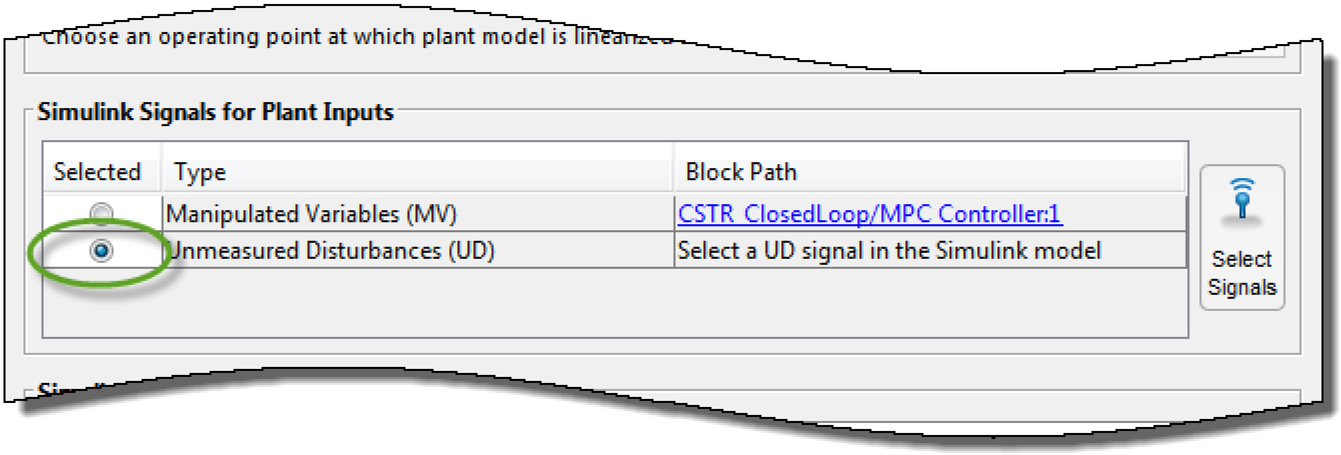
Number of unmeasured outputs: 0

Select Plant Input/Output Signals

Before linearizing the model, assign Simulink signal lines to each MPC signal type in your model. The app uses these signals as linearization inputs and outputs.

In the **Simulink Signals for Plant Inputs** and **Simulink Signals for Plant Outputs** sections, the **Block Path** is automatically defined for manipulated variables, measured outputs, and measured disturbances. **MPC Designer** detects these signals since they are connected to the MPC Controller block. If your application has unmeasured disturbances or unmeasured outputs, select their corresponding Simulink signal lines.

To choose a signal type, use the **Selected** option buttons.



Choose an operating point at which plant model is lineariz

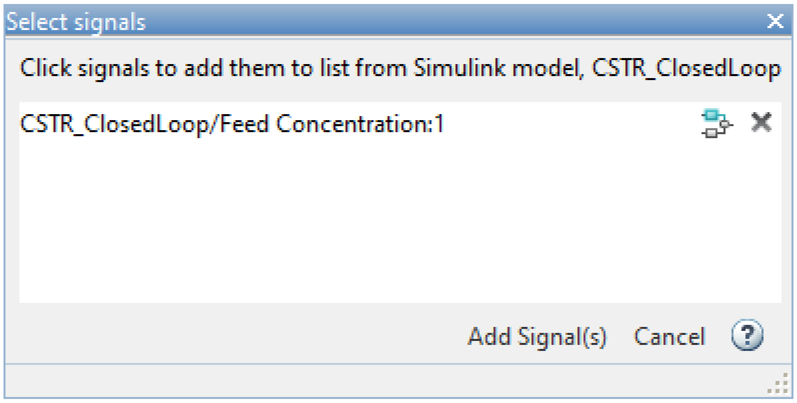
Simulink Signals for Plant Inputs

Selected	Type	Block Path
○	Manipulated Variables (MV)	CSTR_ClosedLoop/MPC Controller:1
●	Unmeasured Disturbances (UD)	Select a UD signal in the Simulink model

Click **Select Signals**.

In the Simulink model window, click the signal line corresponding to the selected signal type.

The signal is highlighted, and its block path is added to the Select signals dialog box.



Select signals

Click signals to add them to list from Simulink model, CSTR_ClosedLoop

CSTR_ClosedLoop/Feed Concentration:1

In the Select signals dialog box, click **Add Signal(s)**.

In the Define MPC Structure By Linearization dialog box, the **Block Path** for the selected signal type updates.

Note If your model has measured disturbances, you must connect the corresponding plant inputs to the signal line connected to the `md` port of the MPC Controller block. For more information, see “Connect Measured Disturbances for Linearization” on page 2-38.

Specify Operating Point

In the **Simulink Operating Point** section, in the drop-down list, select an operating point at which to linearize the model.

For information on the different operating point options, see “Specifying Operating Points” on page 2-29.

Note If you select an option that generates multiple operating points for linearization, **MPC Designer** uses only the first operating point to define the plant structure and linearize the model.

Define Structure and Linearize Model

Click **Define and Linearize**.

The app linearizes the Simulink model at the specified operating point using the specified input/output signals, and adds the linearized plant to the **Data Browser**.

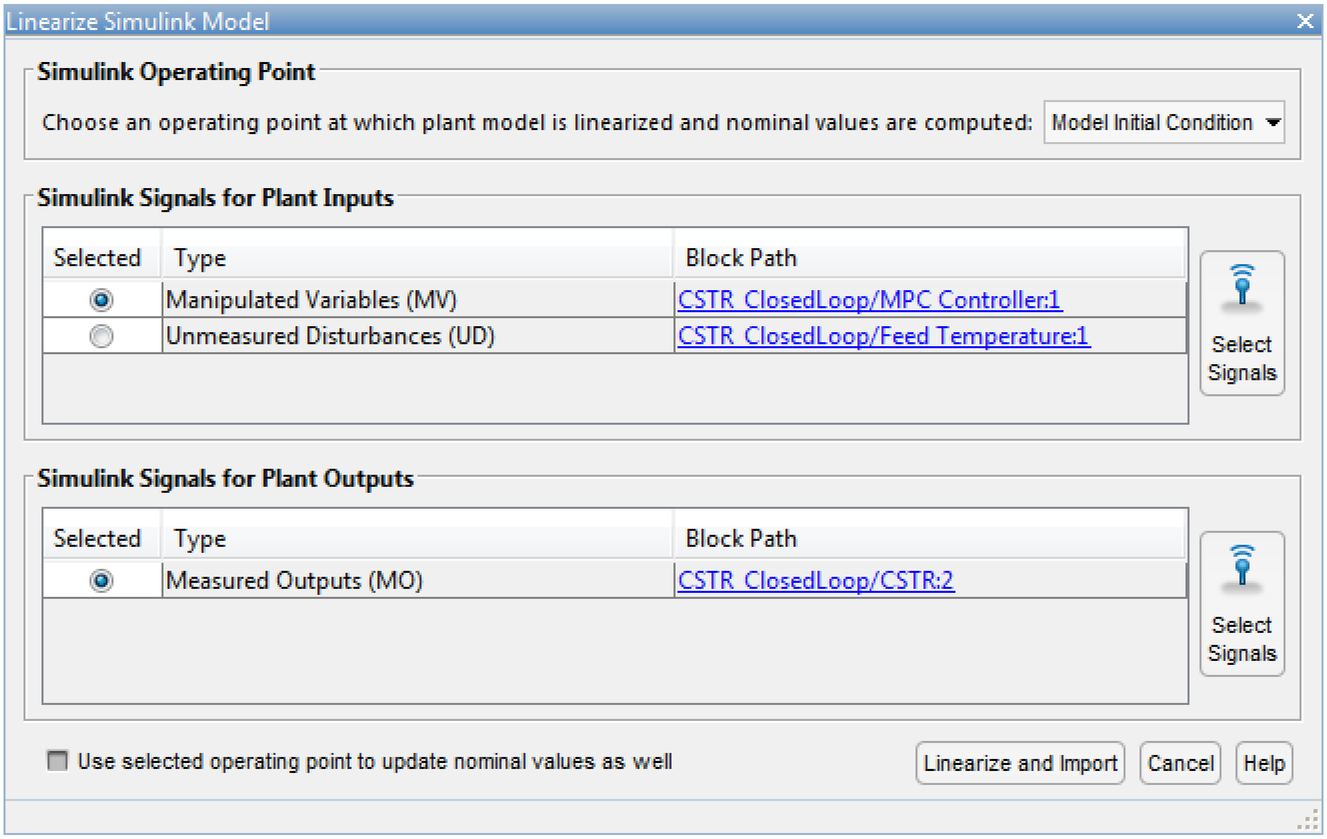
Also, a default controller, which uses the linearized plant as its internal model, and a default simulation scenario are created.

MPC Designer uses the input/output signal values at the selected operating point as nominal values.

Linearize Model

After you define the initial MPC structure, you can linearize the Simulink model at different operating points and import the linearized plants. Doing so is useful for validating controller performance against modeling errors.

On the **MPC Designer** tab, in the **Import** section, click **Linearize Model**.



Simulink Operating Point

Choose an operating point at which plant model is linearized and nominal values are computed: Model Initial Condition

Simulink Signals for Plant Inputs

Selected	Type	Block Path
●	Manipulated Variables (MV)	CSTR_ClosedLoop/MPC Controller:1
○	Unmeasured Disturbances (UD)	CSTR_ClosedLoop/Feed Temperature:1

Simulink Signals for Plant Outputs

Selected	Type	Block Path
●	Measured Outputs (MO)	CSTR_ClosedLoop/CSTR:2

□ Use selected operating point to update nominal values as well

Select Plant Input/Output Signals

In the **Simulink Signals for Plant Inputs** and **Simulink Signals for Plant Outputs** sections, the input/output signal configuration is the same as you specified when initially defining the MPC structure.

You cannot change the signal types and dimensions once the structure has been defined. However, for each signal type, you can select different signal lines from your Simulink model. The selected lines must have the same dimensions as defined in the current MPC structure.

Specify Operating Point

In the **Simulink Operating Point** section, in the drop-down list, select the operating points at which to linearize the model.

For information on the different operating point options, see “Specifying Operating Points” on page 2-29.

Linearize Model and Import Plant

Click **Linearize and Import**.

MPC Designer linearizes the Simulink model at the defined operating point using the specified input/output signals, and adds the linearized plant to the **Data Browser**.

If you select the **Use selected operating point to update nominal values as well** option, the app updates the controller nominal values using the operating point signal values.

If you select an option that generates multiple operating points for linearization, the app linearizes the model at all the specified operating points. The linearized plants are added to the **Data Browser** in the same order in which their corresponding operating points are defined. If you choose to update the nominal values, the app uses the signal values from the first operating point.

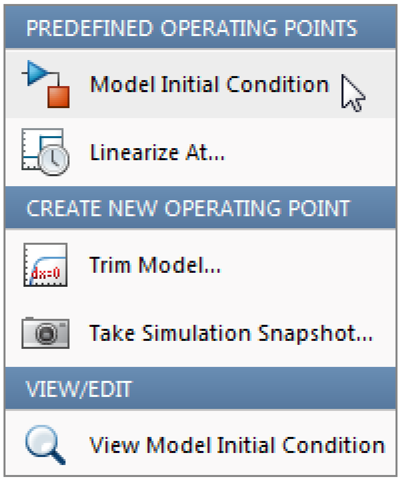
Specifying Operating Points

In the **Simulink Operating Point** section, in the drop-down list, you can select or create operating points for model linearization. For more information on finding steady-state operating points, see “About Operating Points” (Simulink Control Design) and “Compute Steady-State Operating Points from Specifications” (Simulink Control Design).

When using **MPC Designer** in MATLAB Online™, you must linearize your model at the model initial conditions.

Select Model Initial Condition

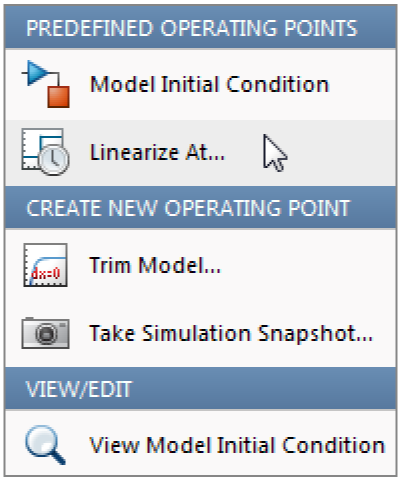
To linearize the model using the initial conditions specified in the Simulink model as the operating point, select **Model Initial Condition**.


PREDEFINED OPERATING POINTS
Model Initial Condition
Linearize At...
CREATE NEW OPERATING POINT
Trim Model...
Take Simulation Snapshot...
VIEW/EDIT
View Model Initial Condition

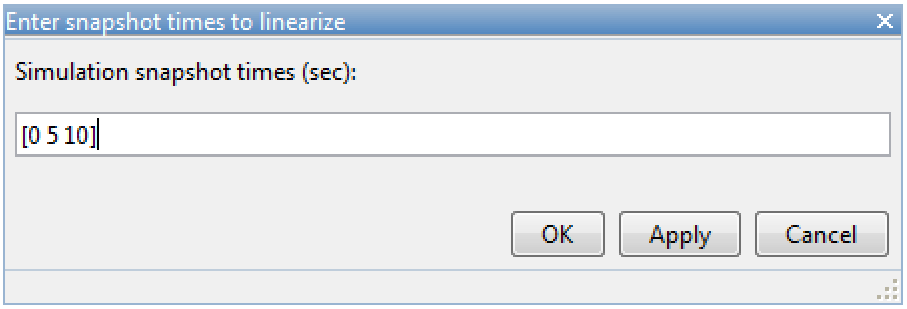
The model initial condition is the default operating point for linearization in **MPC Designer**.

Linearize at Simulation Snapshot Times

To linearize the model at specified simulation snapshot times, select **Linearize At**. Linearizing at snapshot times is useful when you know that your model reaches an equilibrium state after a certain simulation time.


PREDEFINED OPERATING POINTS
Model Initial Condition
Linearize At...
CREATE NEW OPERATING POINT
Trim Model...
Take Simulation Snapshot...
VIEW/EDIT
View Model Initial Condition

In the Enter snapshot times to linearize dialog box, in the **Simulation snapshot times** field, enter one or more simulation snapshot times. Enter multiple snapshot times as a vector.


Enter snapshot times to linearize

Simulation snapshot times (sec): [0 5 10]

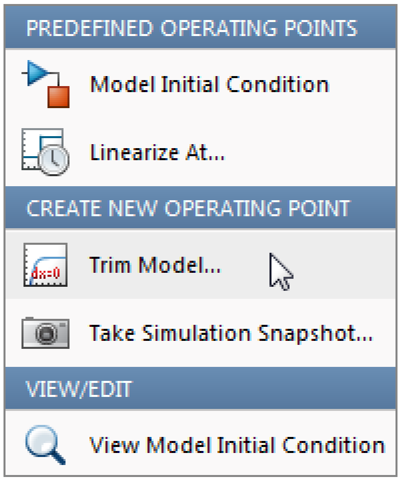
Click **OK**.

If you enter multiple snapshot times, and you selected **Linearize At** from the:

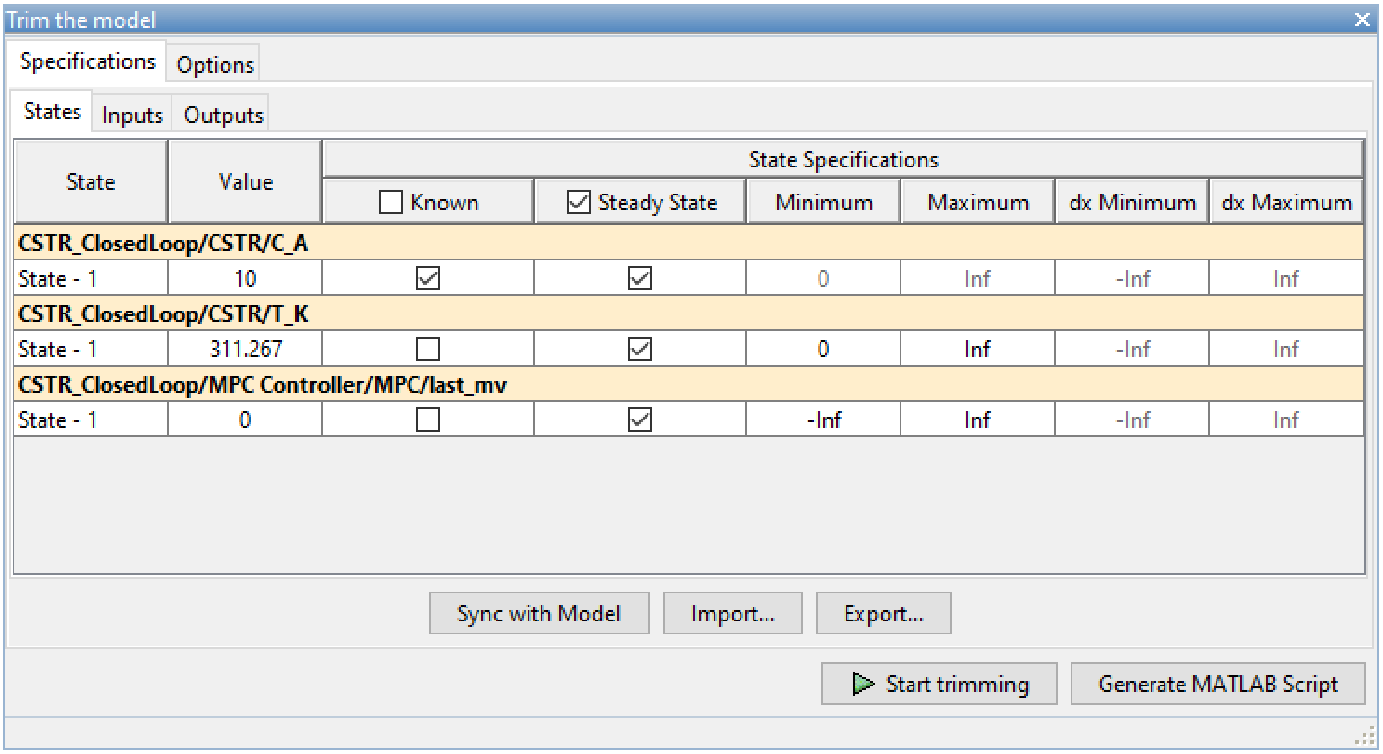
- Define MPC Structure By Linearization dialog box, **MPC Designer** linearizes the model using only the first snapshot time. The nominal values of the MPC controller are defined using the input/output signal values for this snapshot.
- Linearize Simulink Model dialog box, **MPC Designer** linearizes the model at all the specified snapshot times. The linearized plant models are added to the **Data Browser** in the order specified in the snapshot time array. If you selected the **Use selected operating point to update nominal values as well** option, the nominal values are set using the input/output signal values from the first snapshot.

Compute Steady-State Operating Point

To compute a steady-state operating point using numerical optimization methods to meet your specifications, select **Trim Model**.


PREDEFINED OPERATING POINTS
Model Initial Condition
Linearize At...
CREATE NEW OPERATING POINT
Trim Model...
Take Simulation Snapshot...
VIEW/EDIT
View Model Initial Condition

In the Trim the model dialog box, enter the specifications for the steady-state values at which you want to find an operating point. You can specify values for states, input signals, and output signals.



State	Value	State Specifications					
		□ Known	☑ Steady State	Minimum	Maximum	dx Minimum	dx Maximum
CSTR_ClosedLoop/CSTR/C_A							
State - 1	10	☑	☑	0	Inf	-Inf	Inf
CSTR_ClosedLoop/CSTR/T_K							
State - 1	311.267	□	☑	0	Inf	-Inf	Inf
CSTR_ClosedLoop/MPC Controller/MPC/last_mv							
State - 1	0	□	☑	-Inf	Inf	-Inf	Inf

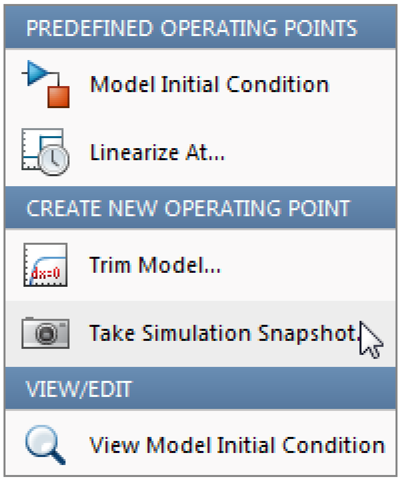
Click **Start Trimming**.

MPC Designer creates an operating point for the given specifications. The computed operating point is added to the **Simulink Operating Point** drop-down list and is selected.

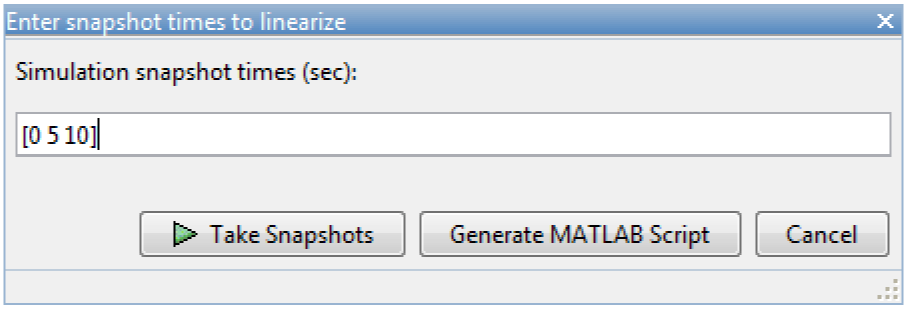
For examples showing how to specify the conditions for a steady-state operating point search, see “Compute Steady-State Operating Points from Specifications” (Simulink Control Design).

Compute Operating Point at Simulation Snapshot Time

To compute operating points using simulation snapshots, select **Take Simulation Snapshot**. Linearizing the model using operating points computed from simulation snapshots is useful when you know that your model reaches an equilibrium state after a certain simulation time.



In the Enter snapshot times to linearize dialog box, in the **Simulation snapshot times** field, enter one or more simulation snapshot times. Enter multiple snapshot times as a vector.



Enter snapshot times to linearize

Simulation snapshot times (sec): [0 5 10]

Click **Take Snapshots**.

MPC Designer simulates the Simulink model. At each snapshot time, the current state of the model is used to create an operating point, which is added to the drop-down list and selected.

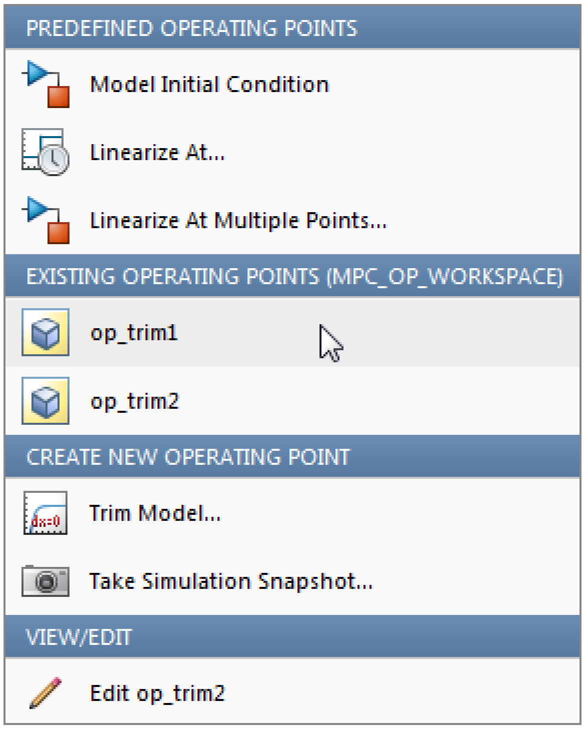
If you entered multiple snapshot times, the operating points are stored together as an array. If you selected **Take Simulation Snapshot** from the:

- Define MPC Structure By Linearization dialog box, **MPC Designer** linearizes the model using only the first operating point in the array. The nominal values of the MPC controller are defined using the input/output signal values for this operating point.
- Linearize Simulink Model dialog box, **MPC Designer** linearizes the model at all the operating points in the array. The linearized plant models are added to the **Data Browser** in the same order as the operating point array.

In **MPC Designer**, the **Linearize At** and **Take Simulation Snapshot** options generally produce the same linearized plant and nominal signal values. However, since the **Take Simulation Snapshot** option first computes an operating point from the snapshot before linearization, the results can differ.

Select Existing Operating Point

Under **Existing Operating Points**, select a previously defined operating point at which to linearize the Simulink model. This option is available if one or more previously created operating points are available in the drop-down list.

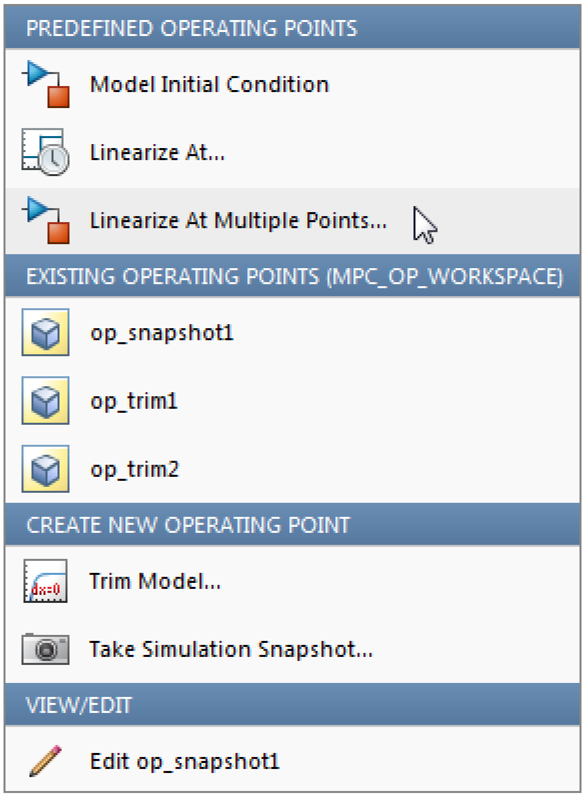

PREDEFINED OPERATING POINTS
Model Initial Condition
Linearize At...
Linearize At Multiple Points...
EXISTING OPERATING POINTS (MPC_OP_WORKSPACE)
op_trim1
op_trim2
CREATE NEW OPERATING POINT
Trim Model...
Take Simulation Snapshot...
VIEW/EDIT
Edit op_trim2

If the selected operating point represents an operating point array created using multiple snapshot times, and you selected an operating point from the:

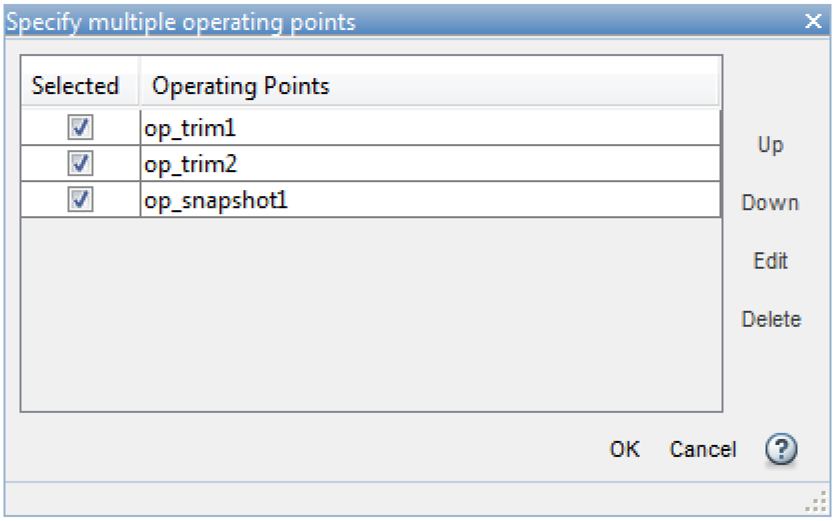
- Define MPC Structure By Linearization dialog box, **MPC Designer** linearizes the model using only the first operating point in the array. The nominal values of the MPC controller are defined using the input/output signal values for this operating point.
- Linearize Simulink Model dialog box, **MPC Designer** linearizes the model at all the operating points in the array. The linearized plant models are added to the **Data Browser** in the same order as the operating point array.

Select Multiple Operating Points

To linearize the Simulink model at multiple existing operating points, select **Linearize at Multiple Points**. This option is available if there are more than one previously created operating points in the drop-down list.


PREDEFINED OPERATING POINTS
Model Initial Condition
Linearize At...
Linearize At Multiple Points...
EXISTING OPERATING POINTS (MPC_OP_WORKSPACE)
op_snapshot1
op_trim1
op_trim2
CREATE NEW OPERATING POINT
Trim Model...
Take Simulation Snapshot...
VIEW/EDIT
Edit op_snapshot1

In the Specify multiple operating points dialog box, select the operating points at which to linearize the model.



Selected	Operating Points
☑	op_trim1
☑	op_trim2
☑	op_snapshot1

To change the operating point order, click an operating point in the list and click **Up** or **Down** to move the highlighted operating point within the list.

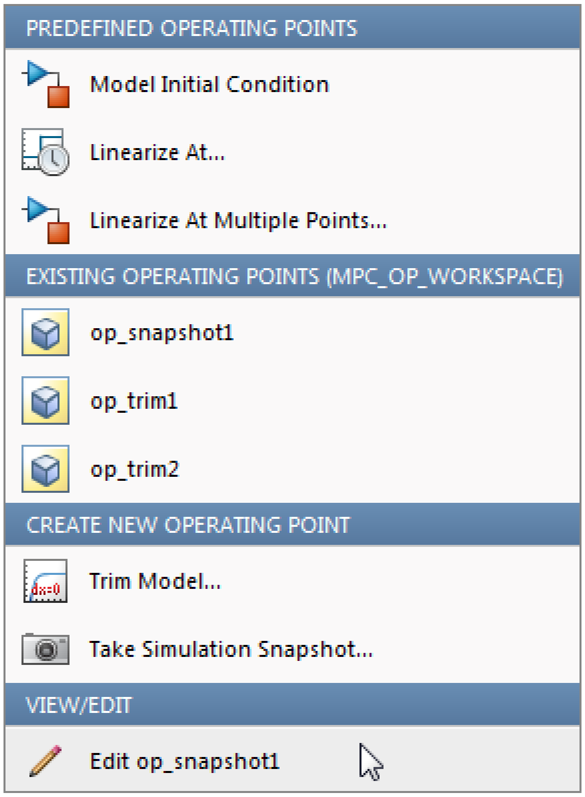
Click **OK**.

If you selected **Linearize at Multiple Points** from the:

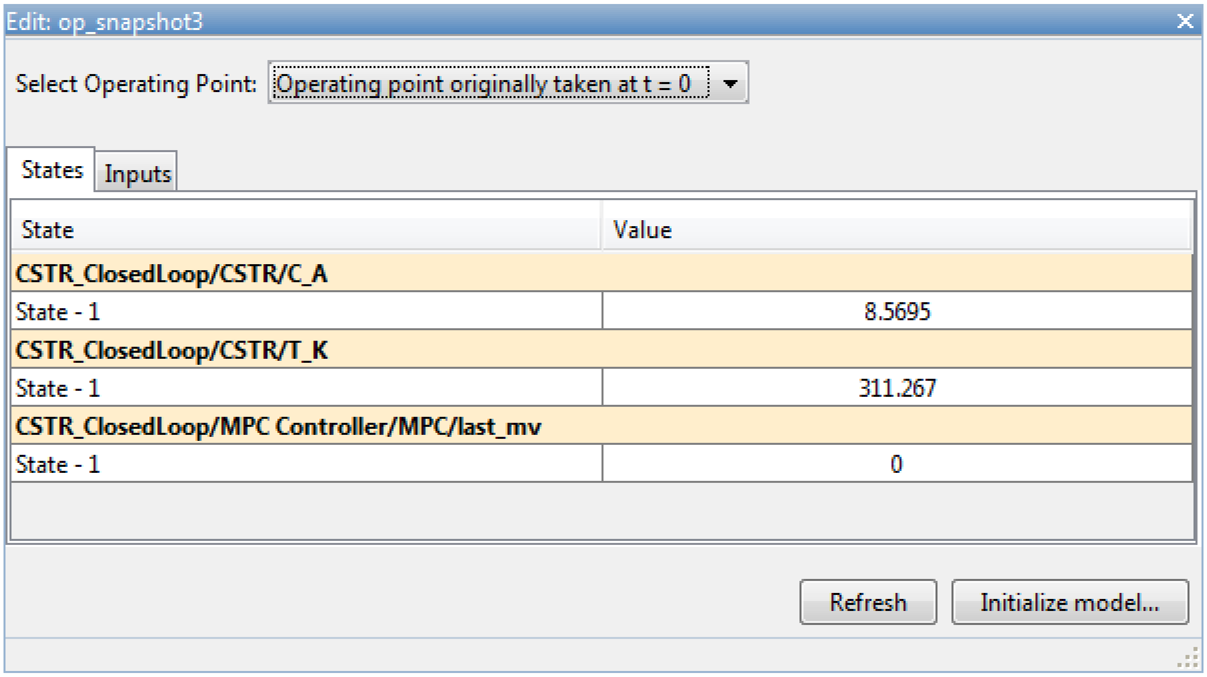
- Define MPC Structure By Linearization dialog box, **MPC Designer** linearizes the model using only the first specified operating point. The nominal values of the MPC controller are defined using the input/output signal values for this operating point.
- Linearize Simulink Model dialog box, **MPC Designer** linearizes the model at all the specified operating points. The linearized plant models are added to the **Data Browser** in the order specified in the Specify multiple operating points dialog box.

View/Edit Operating Point

To view or edit the selected operating point, under **View/Edit**, click the **Edit** option.


PREDEFINED OPERATING POINTS
Model Initial Condition
Linearize At...
Linearize At Multiple Points...
EXISTING OPERATING POINTS (MPC_OP_WORKSPACE)
op_snapshot1
op_trim1
op_trim2
CREATE NEW OPERATING POINT
Trim Model...
Take Simulation Snapshot...
VIEW/EDIT
Edit op_snapshot1

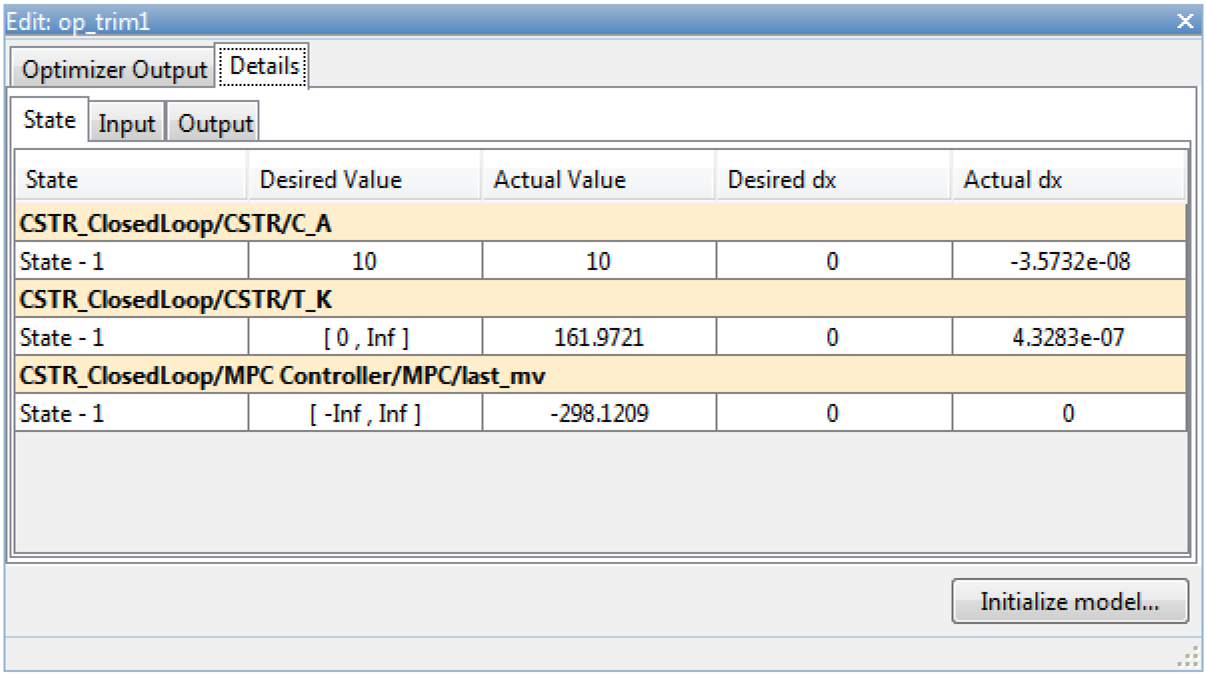
In the Edit dialog box, if you created the selected operating point from a simulation snapshot, you can edit the operating point values.



State	Value
CSTR_ClosedLoop/CSTR/C_A	
State - 1	8.5695
CSTR_ClosedLoop/CSTR/T_K	
State - 1	311.267
CSTR_ClosedLoop/MPC Controller/MPC/last_mv	
State - 1	0

If the selected operating point represents an operating point array, in the **Select Operating Point** drop-down list, select an operating point to view.

If you obtained the operating point by trimming the model, you can only view the operating point values.



State	Desired Value	Actual Value	Desired dx	Actual dx
CSTR_ClosedLoop/CSTR/C_A				
State - 1	10	10	0	-3.5732e-08
CSTR_ClosedLoop/CSTR/T_K				
State - 1	[0 , Inf]	161.9721	0	4.3283e-07
CSTR_ClosedLoop/MPC Controller/MPC/last_mv				
State - 1	[-Inf , Inf]	-298.1209	0	0

To set the Simulink model initial conditions to the states in the operating point, click **Initialize model**. You can then simulate the model at the specified operating point.

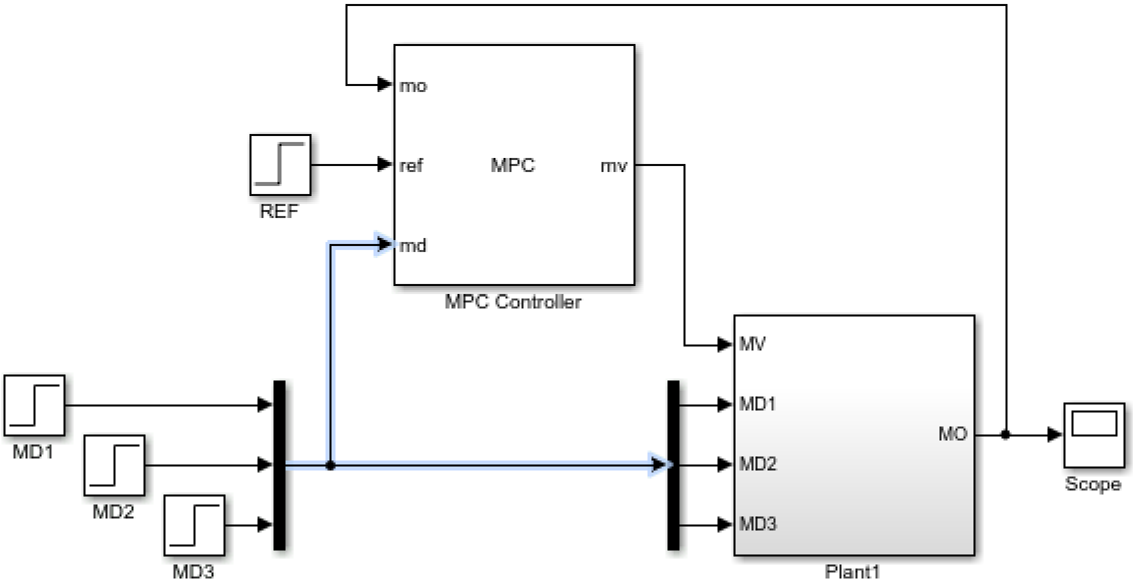
When setting the model initial conditions, **MPC Designer** exports the operating point to the MATLAB workspace. Also, in the Simulink Configuration Parameters dialog box, in the **Data Import/Export** section, it selects the **Input** and **Initial state** parameters and configures them to use the states and inputs in the exported operating point.

To reset the model initial conditions, for example if you delete the exported operating point, clear the **Input** and **Initial state** parameters.

Connect Measured Disturbances for Linearization

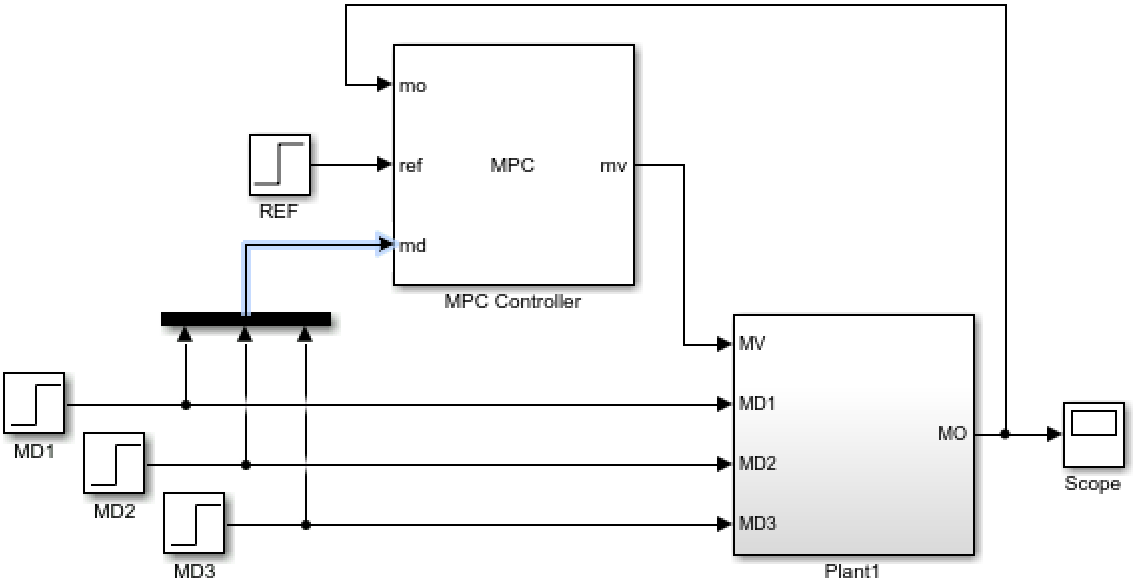
If your Simulink model has measured disturbance signals, connect them to the corresponding plant input ports and to the `md` port of the MPC Controller block. If you have multiple measured disturbances, connect them to the MPC Controller using a vector signal. As discussed in “Define MPC Structure By Linearization” on page 2-23, **MPC Designer** automatically detects the measured disturbances connected to the MPC Controller block and sets them as plant inputs for linearization.

Since the measured disturbances connected to the `md` port are selected as linearization inputs, you must connect the plant measured disturbance input ports to the selected signal line, as shown in the following:



Correct MD Connection

If you connect the plant measured disturbance input ports to the corresponding signals before the Mux block, as shown in the following, there is no linearization path from the signals at the `md` port to the plant. As a result, when you linearize the plant using **MPC Designer**, the measured disturbance channels linearize to zero.



Incorrect MD Connection

See Also

MPC Designer

Related Examples

- “Linearize Simulink Models” on page 2-16
- “Design MPC Controller in Simulink” on page 3-31

Identify Plant from Data

When designing a model predictive controller, you can specify the internal predictive plant model using a linear identified model. You use System Identification Toolbox software to estimate a linear plant model in one of these forms:

- State-space model — `idss`
- Transfer function model — `idtf`
- Polynomial model — `idpoly`
- Process model — `idproc`
- Grey-box model — `idgrey`

You can estimate the plant model programmatically at the command line or interactively using the **System Identification** app.

Identify Plant from Data at the Command Line

This example shows how to identify a plant model at the command line. For information on identifying models using the System Identification app, see “Identify Linear Models Using System Identification App” (System Identification Toolbox).

Load the measured input/output data.

```
load plantIO
```

This command imports the plant input signal, `u`, plant output signal, `y`, and sample time, `Ts` to the MATLAB® workspace.

Create an `iddata` object from the input and output data.

```
mydata = iddata(y,u,Ts);
```

You can optionally assign channel names and units for the input and output signals.

```
mydata.InputName = 'Voltage';
mydata.InputUnit = 'V';
mydata.OutputName = 'Position';
mydata.OutputUnit = 'cm';
```

Typically, you must preprocess identification I/O data before estimating a model. For this example, remove the offsets from the input and output signals by detrending the data.

```
mydatad = detrend(mydata);
```

You can also remove offsets by creating an `ssestOptions` object and specifying the `InputOffset` and `OutputOffset` options.

For this example, estimate a second-order, linear state-space model using the detrended data. To estimate a discrete-time model, specify the sample time as `Ts`.

```
ss1 = ssest(mydatad,2,'Ts',Ts)
```

```
ss1 =
  Discrete-time identified state-space model:
    x(t+Ts) = A x(t) + B u(t) + K e(t)
       y(t) = C x(t) + D u(t) + e(t)

  A = 
            x1       x2
   x1   0.8942  -0.1575
   x2   0.1961   0.7616

  B = 
         Voltage
   x1  6.008e-05
   x2   -0.01219

  C = 
                  x1       x2
   Position    38.24  -0.3835

  D = 
             Voltage
   Position        0

  K = 
       Position
   x1   0.03572
   x2    0.0223

Sample time: 0.1 seconds

Parameterization:
   FREE form (all coefficients in A, B, C free).
   Feedthrough: none
   Disturbance component: estimate
   Number of free coefficients: 10
   Use "idssdata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using SSEST on time domain data "mydatad".
Fit to estimation data: 89.85% (prediction focus)
FPE: 0.0156, MSE: 0.01541
```

You can use this identified plant as the internal prediction model for your MPC controller. When you do so, the controller converts the identified model to a discrete-time, state-space model.

By default, the MPC controller discards any unmeasured noise components from your identified model. To configure noise channels as unmeasured disturbances, you must first create an augmented state-space model from your identified model. For example:

```
ss2 = ss(ss1,'augmented')
```

```
ss2 =

  A = 
            x1       x2
   x1   0.8942  -0.1575
   x2   0.1961   0.7616

  B = 
          Voltage  v@Position
   x1   6.008e-05    0.004448
   x2    -0.01219    0.002777

  C =
                  x1       x2
   Position    38.24  -0.3835

  D =
                Voltage  v@Position
   Position           0      0.1245

Input groups:
      Name      Channels
    Measured       1
     Noise         2

Sample time: 0.1 seconds
Discrete-time state-space model.
```

This command creates a state-space model, `ss2`, with two input groups, `Measured` and `Noise`, for the measured and noise inputs respectively. When you import the augmented model into your MPC controller, channels in the `Noise` input group are defined as unmeasured disturbances.

Working with Impulse-Response Models

You can use System Identification Toolbox software to estimate finite step-response or finite impulse-response (FIR) plant models using measured data. Such models, also known as nonparametric models, are easy to determine from plant data ([1] and [2]) and have intuitive appeal.

Use the `impulseest` function to estimate an FIR model from measured data. This function generates the FIR coefficients encapsulated as an `idtf` object; that is, a transfer function model with only numerator coefficients. `impulseest` is especially effective in situations where the input signal used for identification has low excitation levels. To design a model predictive controller for this plant, you can convert the identified FIR plant model to a numeric LTI model. However, this conversion usually yields a high-order plant, which can degrade the controller design. For example, the numerical precision issues with high-order plants can affect estimator design. This result is particularly an issue for MIMO systems.

Model predictive controllers work best with low-order parametric models. Therefore, to design a model predictive controller using measured plant data, you can:

- Estimate a low-order parametric model using a parametric estimator, such as `ssest`.
- Initially identify a nonparametric model using `impulseest`, and then estimate a low-order parametric model from the response of the nonparametric model. For an example, see [3].
- Initially identify a nonparametric model using `impulseest`, and then convert the FIR model to a state-space model using `idss`. You can then reduce the order of the state-space model using `balred`. This approach is similar to the method used by `ssregest`.

References

[1] Cutler, C., and F. Yocum, "Experience with the DMC inverse for identification," *Chemical Process Control — CPC IV* (Y. Arkun and W. H. Ray, eds.), CACHE, 1991.

[2] Ricker, N. L., "The use of bias least-squares estimators for parameters in discrete-time pulse response models," *Ind. Eng. Chem. Res.*, Vol. 27, pp. 343, 1988.

[3] Wang, L., P. Gawthrop, C. Chessari, T. Podsiadly, and A. Giles, "Indirect approach to continuous time system identification of food extruder," *J. Process Control*, Vol. 14, Number 6, pp. 603–615, 2004.

See Also

Apps
System Identification

Functions
`detrend` | `iddata` | `ssest`

More About

- “Handling Offsets and Trends in Data” (System Identification Toolbox)
- “Identify Linear Models Using System Identification App” (System Identification Toolbox)
- “Design MPC Controller for Identified Plant Model”

Design MPC Controllers

Design Controller Using MPC Designer

This example shows how to design a model predictive controller for a continuous stirred-tank reactor (CSTR) using **MPC Designer**.

CSTR Model

The following differential equations represent the linearized model of a continuous stirred-tank reactor (CSTR) involving an exothermic reaction:

$$\frac{dC'_A}{dt} = a_{11}C'_A + a_{12}T' + b_{11}T'_c + b_{12}C'_{Ai}$$

$$\frac{dT'}{dt} = a_{21}C'_A + a_{22}T' + b_{21}T'_c + b_{22}C'_{Ai}$$

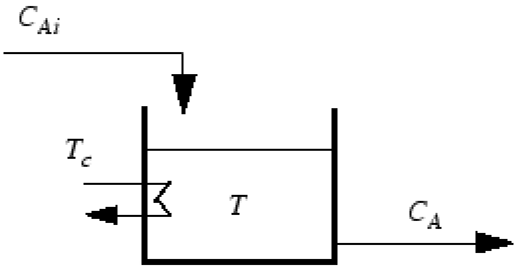
where the inputs are:

- C_{Ai} — Concentration of reagent A in the feed stream (kgmol/m^3)
- T_c — Reactor coolant temperature (degrees C)

and the outputs are:

- T — Reactor temperature (degrees C)
- C_A — Residual concentration of reagent A in the product stream (kgmol/m^3)

The prime terms, such as C'_A, denote a deviation from the nominal steady-state condition at which the model has been linearized.



Measurement of reagent concentrations is often difficult. For this example, assume that:

- T_c is a manipulated variable.
- C_{Ai} is an unmeasured disturbance.
- T is a measured output.
- C_A is an unmeasured output.

The model can be described in state-space format:

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where,

$$x = \begin{bmatrix} C'_A \\ T' \end{bmatrix}, u = \begin{bmatrix} T'_c \\ C'_{Ai} \end{bmatrix}, y = \begin{bmatrix} T' \\ C'_A \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

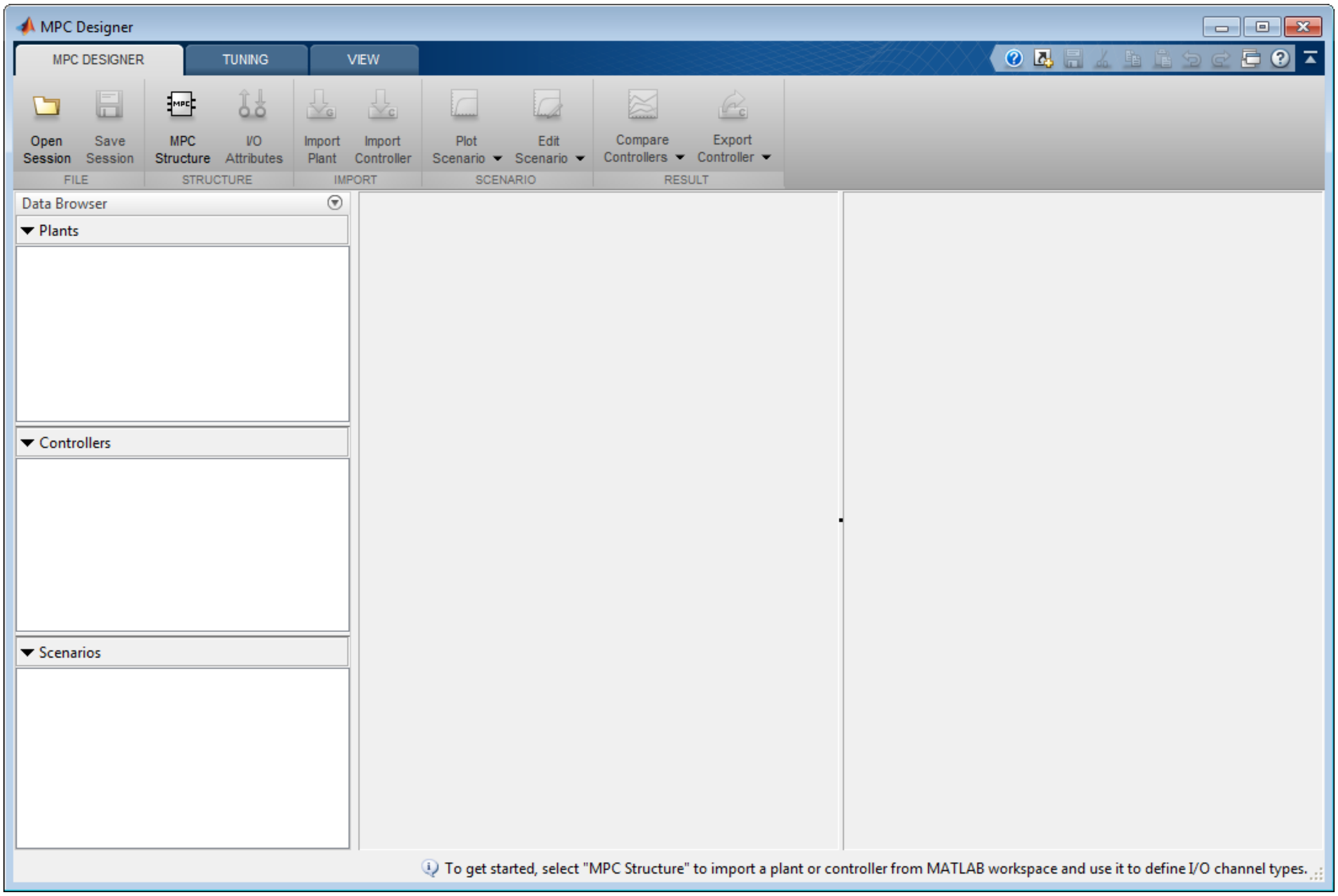
For this example, the coolant temperature has a limited range of ±10 degrees from its nominal value and a limited rate of change of ±4 degrees per sample period.

Create a state-space model of a CSTR system.

```
A = [-0.0285 -0.0014; -0.0371 -0.1476];
B = [-0.0850 0.0238; 0.0802 0.4462];
C = [0 1; 1 0];
D = zeros(2,2);
CSTR = ss(A,B,C,D);
```

Import Plant and Define MPC Structure

```
mpcDesigner
```



To get started, select "MPC Structure" to import a plant or controller from MATLAB workspace and use it to define I/O channel types.

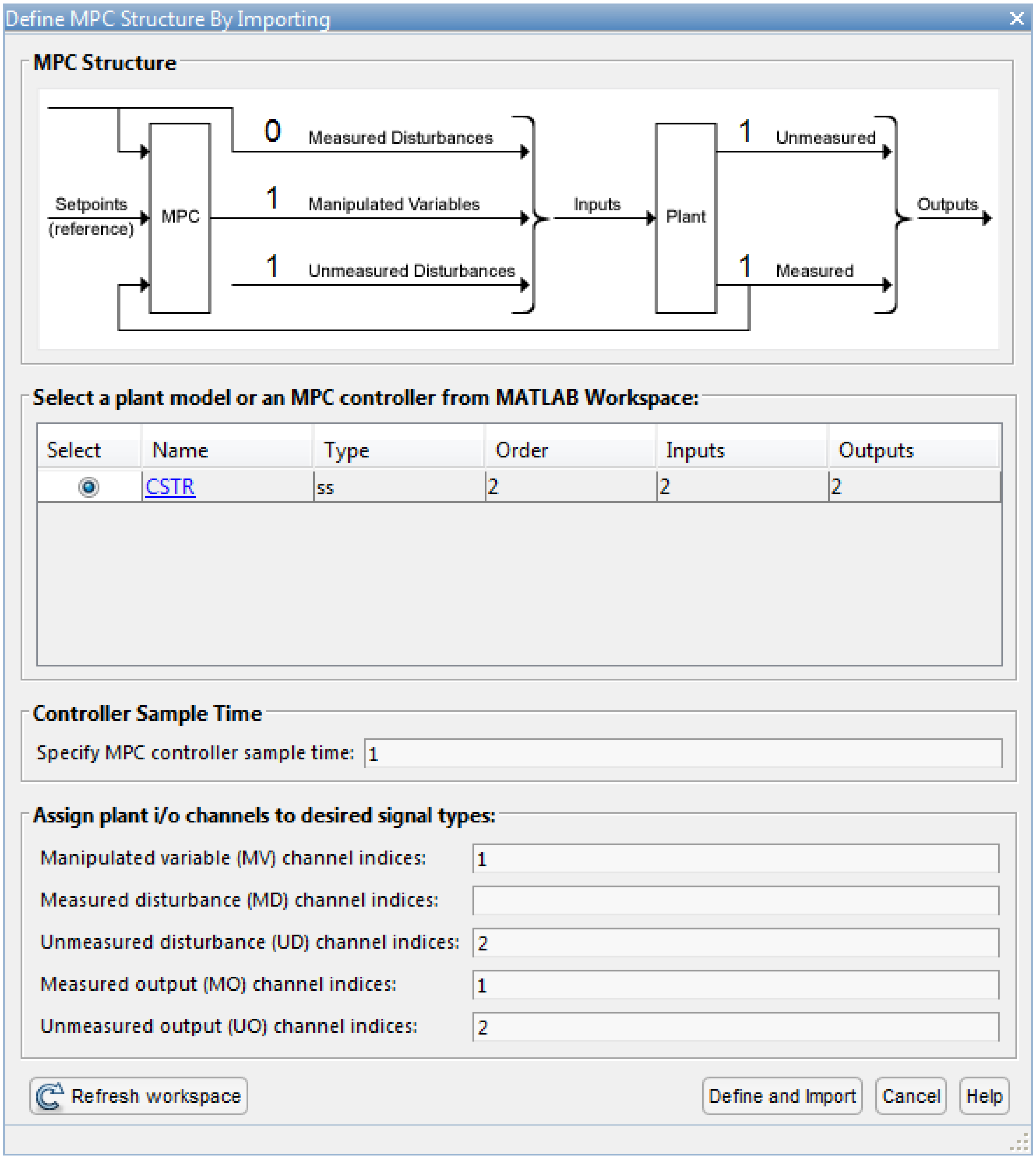
On the **MPC Designer** tab, in the **Structure** section, click **MPC Structure**.

In the Define MPC Structure By Importing dialog box, in the **Select a plant model or an MPC controller** table, select the CSTR model.

Since CSTR is a stable, continuous-time LTI system, **MPC Designer** sets the controller sample time to 0.1 T_r, where T_r is the average rise time of CSTR. For this example, in the **Specify MPC controller sample time** field, enter a sample time of 1.

By default, all plant inputs are defined as manipulated variables and all plant outputs as measured outputs. In the **Assign plant i/o channels** section, assign the input and output channel indices such that:

- The first input, coolant temperature, is a manipulated variable.
- The second input, feed concentration, is an unmeasured disturbance.
- The first output, reactor temperature, is a measured output.
- The second output, reactant concentration, is an unmeasured output.



Define MPC Structure By Importing

MPC Structure

Select a plant model or an MPC controller from MATLAB Workspace:

Select	Name	Type	Order	Inputs	Outputs
●	CSTR	ss	2	2	2

Controller Sample Time

Specify MPC controller sample time: 1

Assign plant i/o channels to desired signal types:

Manipulated variable (MV) channel indices: 1

Measured disturbance (MD) channel indices:

Unmeasured disturbance (UD) channel indices: 2

Measured output (MO) channel indices: 1

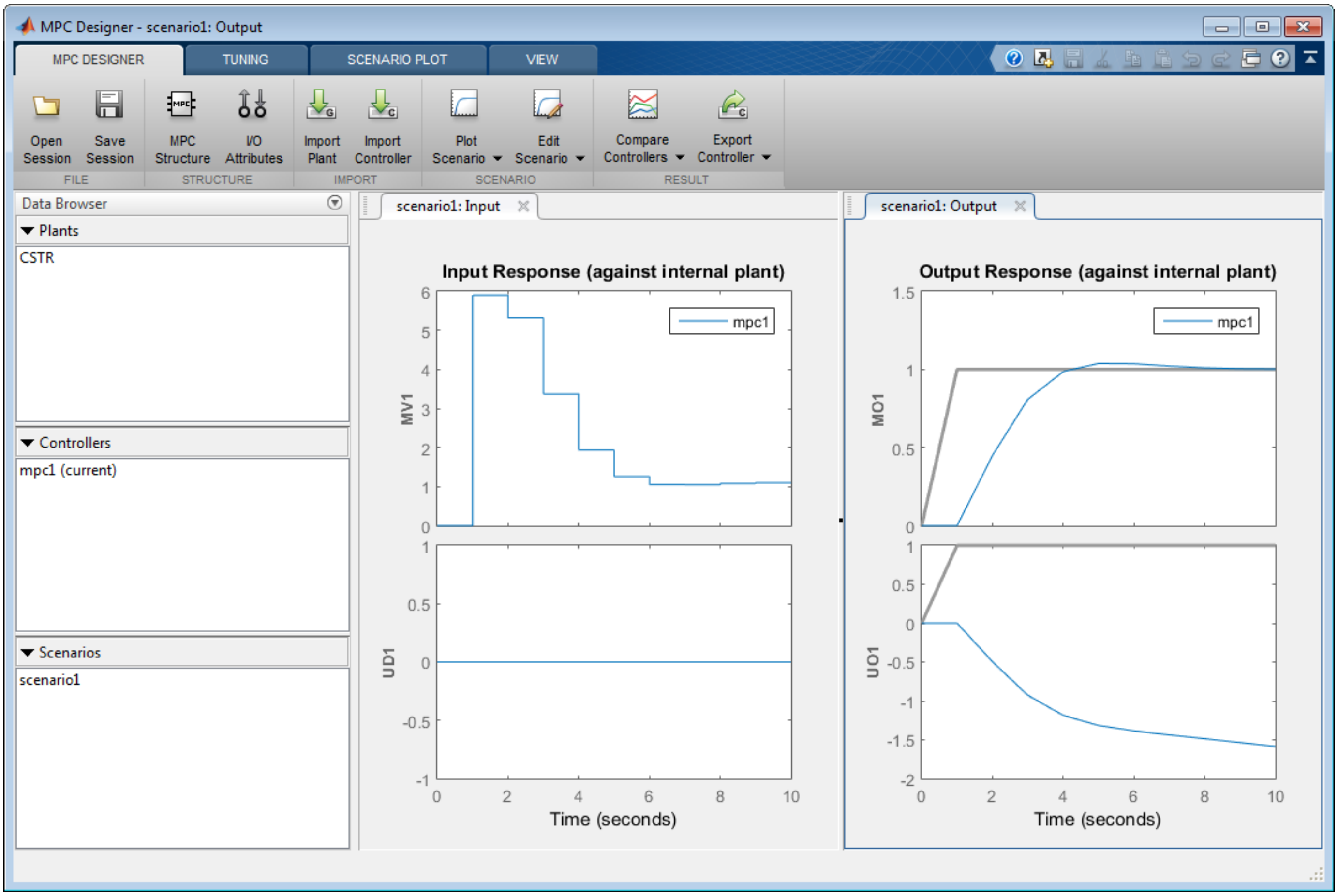
Unmeasured output (UO) channel indices: 2

Click **Define and Import**.

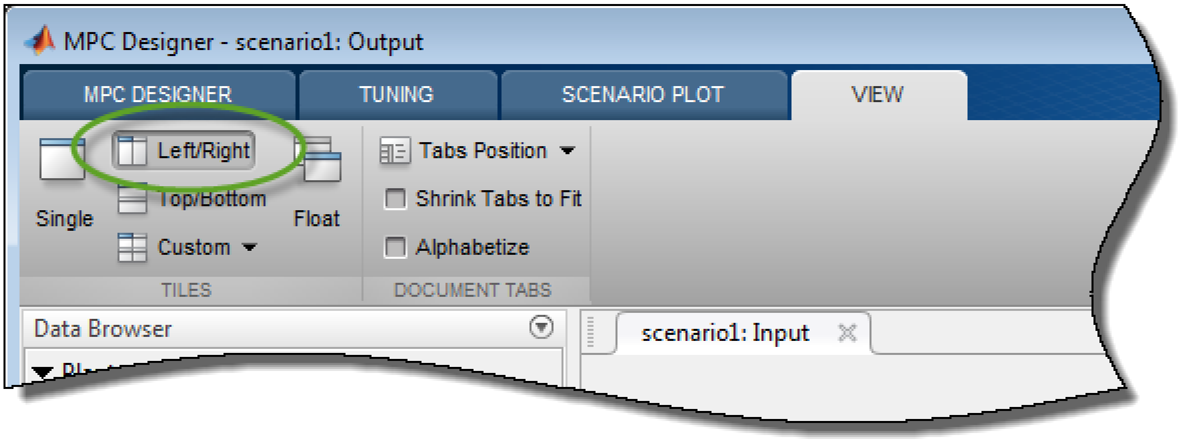
The app imports the `CSTR` plant to the **Data Browser**. The following are also added to the **Data Browser**:

- `mpc1` — Default MPC controller created using `sys` as its internal model.
- `scenario1` — Default simulation scenario.

The app runs the default simulation scenario and updates the **Input Response** and **Output Response** plots.



Tip To view the response plots side-by-side, on the **View** tab, in the **Tiles** section, click **Left/Right**.



Once you define the MPC structure, you cannot change it within the current **MPC Designer** session. To use a different channel configuration, start a new session of the app.

Define Input and Output Channel Attributes

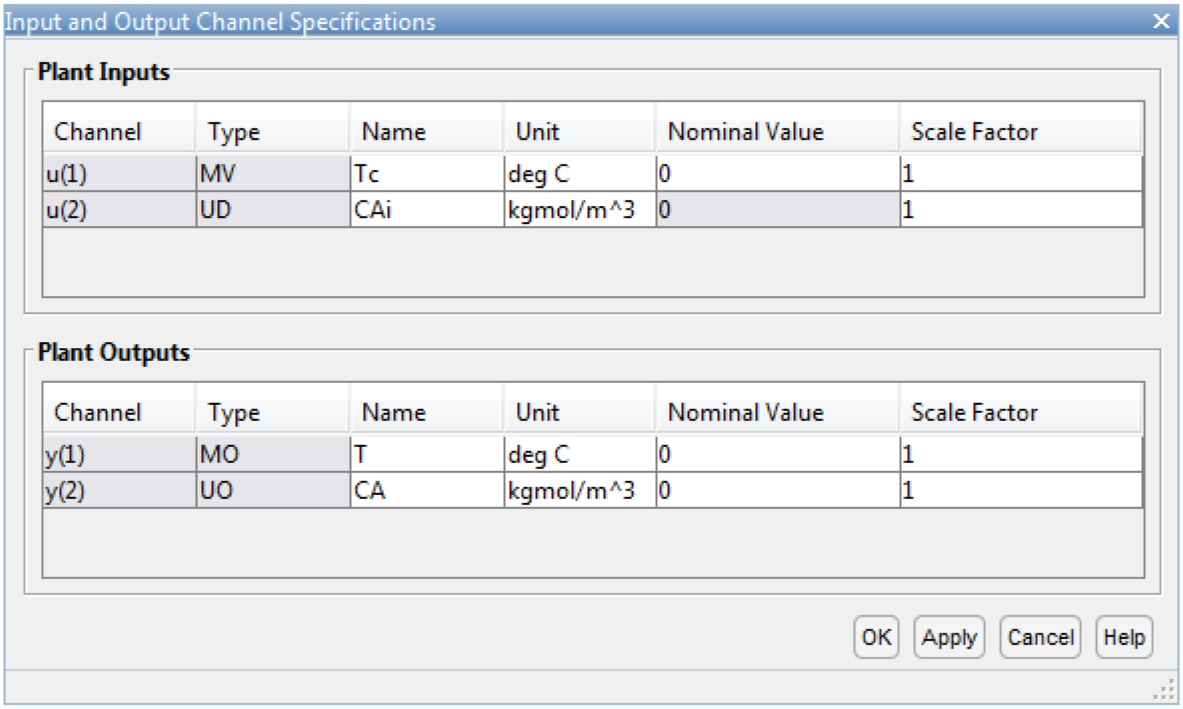
On the **MPC Designer** tab, select **I/O Attributes**.

In the Input and Output Channel Specifications dialog box, in the **Name** column, specify a meaningful name for each input and output channel.

In the **Unit** column, optionally specify the units for each channel.

Since the state-space model is defined using deviations from the nominal operating point, set the **Nominal Value** for each input and output channel to `0`.

Keep the **Scale Factor** for each channel at the default value of `1`.



Plant Inputs

Channel	Type	Name	Unit	Nominal Value	Scale Factor
u(1)	MV	Tc	deg C	0	1
u(2)	UD	CAi	kgmol/m^3	0	1

Plant Outputs

Channel	Type	Name	Unit	Nominal Value	Scale Factor
y(1)	MO	T	deg C	0	1
y(2)	UO	CA	kgmol/m^3	0	1

Click **OK**.

The **Input Response** and **Output Response** plot labels update to reflect the new signal names and units.

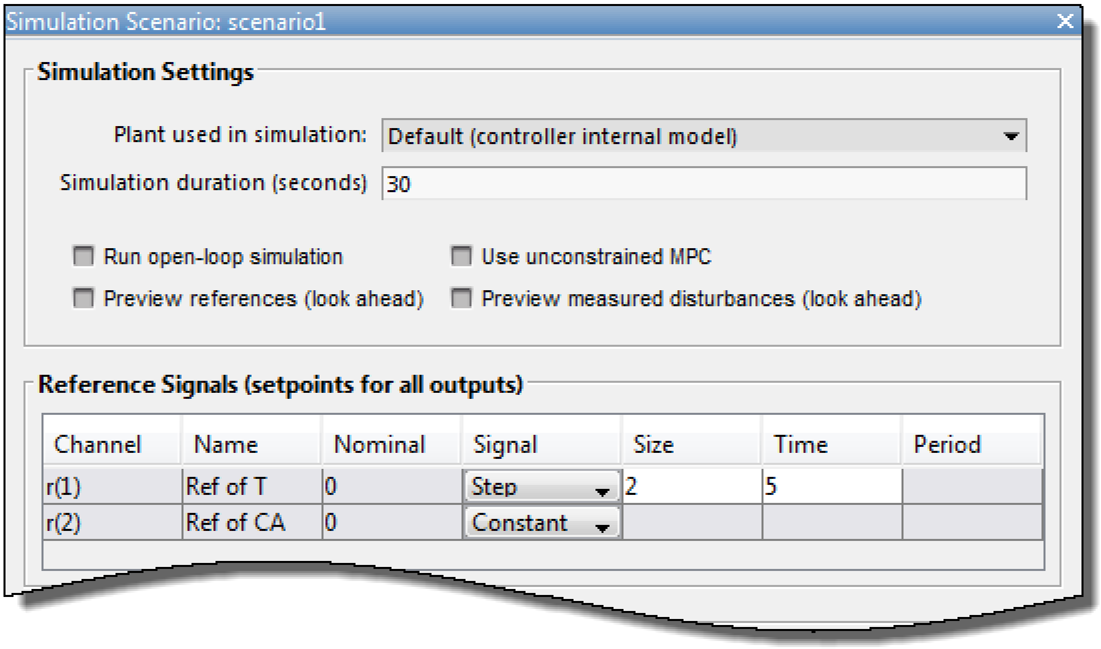
Configure Simulation Scenario

On the **MPC Designer** tab, in the **Scenario** section, click **Edit Scenario** > **scenario1**.

In the Simulation Scenario dialog box, increase the **Simulation duration** to `30` seconds.

In the **Reference Signals** table, in the first row, specify a step **Size** of `2` and a **Time** of `5`.

In the **Signal** column, in the second row, select a `Constant` reference to hold the concentration setpoint at its nominal value.



Simulation Scenario: scenario1

Simulation Settings

Plant used in simulation: Default (controller internal model)

Simulation duration (seconds) 30

□ Run open-loop simulation □ Use unconstrained MPC

□ Preview references (look ahead) □ Preview measured disturbances (look ahead)

Reference Signals (setpoints for all outputs)

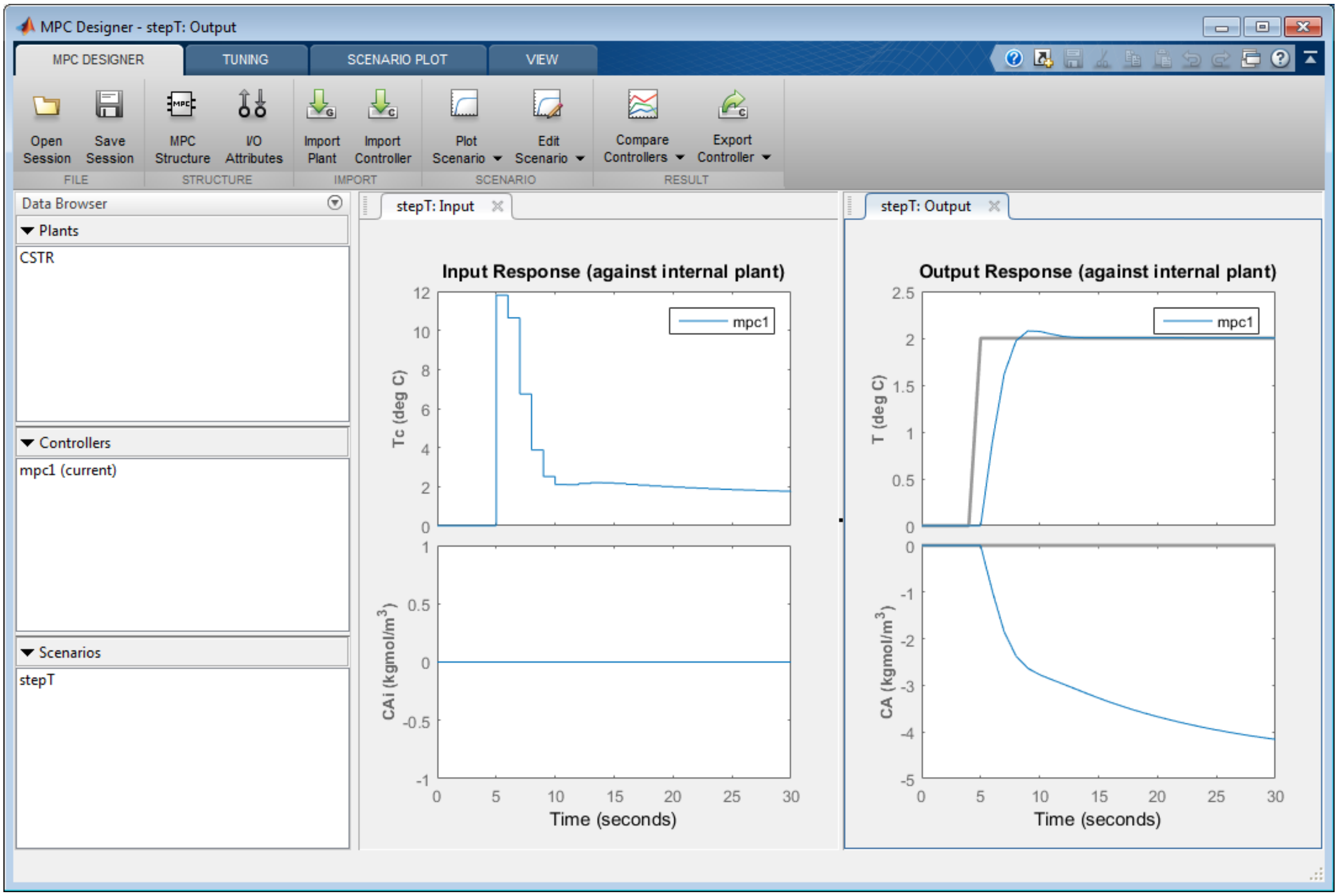
Channel	Name	Nominal	Signal	Size	Time	Period
r(1)	Ref of T	0	Step	2	5	
r(2)	Ref of CA	0	Constant			

The default scenario is configured to simulate a step change of 2 degrees in the reactor temperature, *T*, at a time of 5 seconds.

Click **OK**.

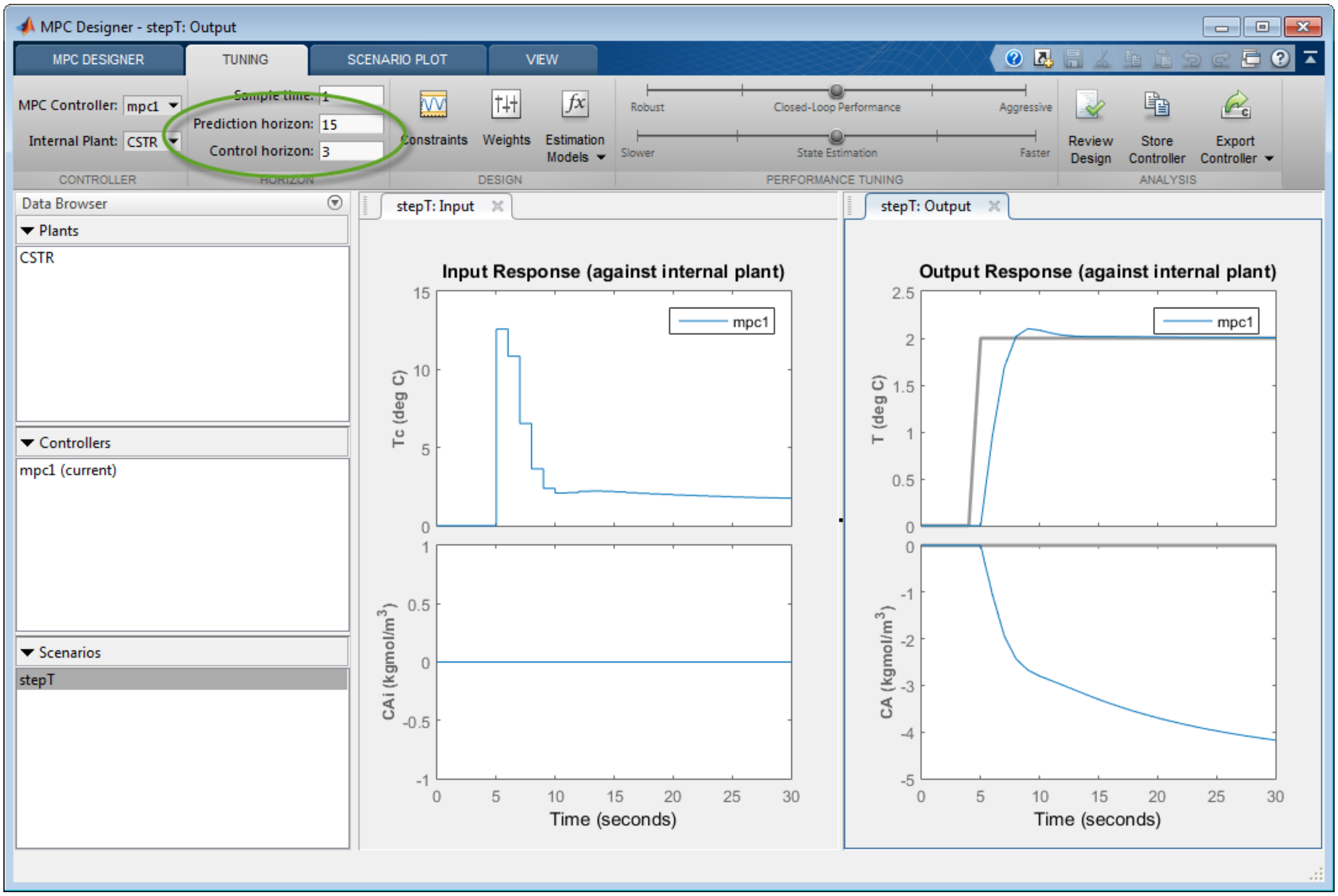
The response plots update to reflect the new simulation scenario configuration.

In the **Data Browser**, in the **Scenarios** section, click `scenario1`. Click `scenario1` a second time, and rename the scenario to `stepT`.



Configure Controller Horizons

On the **Tuning** tab, in the **Horizons** section, specify a **Prediction horizon** of 15 and a **Control horizon** of 3.



MPC Controller: mpc1

Internal Plant: CSTR

Sample time: 1

Prediction horizon: 15

Control horizon: 3

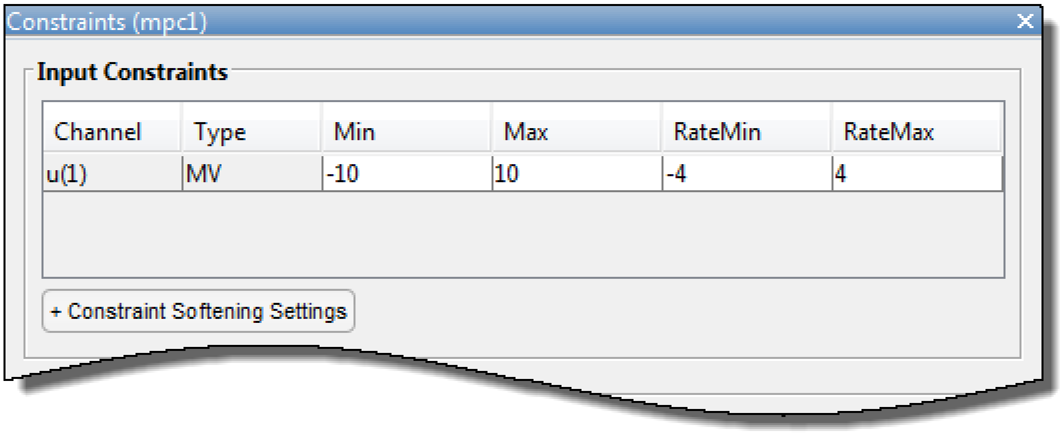
The response plots update to reflect the new horizons. The **Input Response** plot shows that the control actions for the manipulated variable violate the required coolant temperature constraints.

Define Input Constraints

In the **Design** section, click **Constraints**.

In the Constraints dialog box, in the **Input Constraints** section, enter the coolant temperature upper and lower bounds in the **Min** and **Max** columns respectively.

Specify the rate of change limits in the **RateMin** and **RateMax** columns.



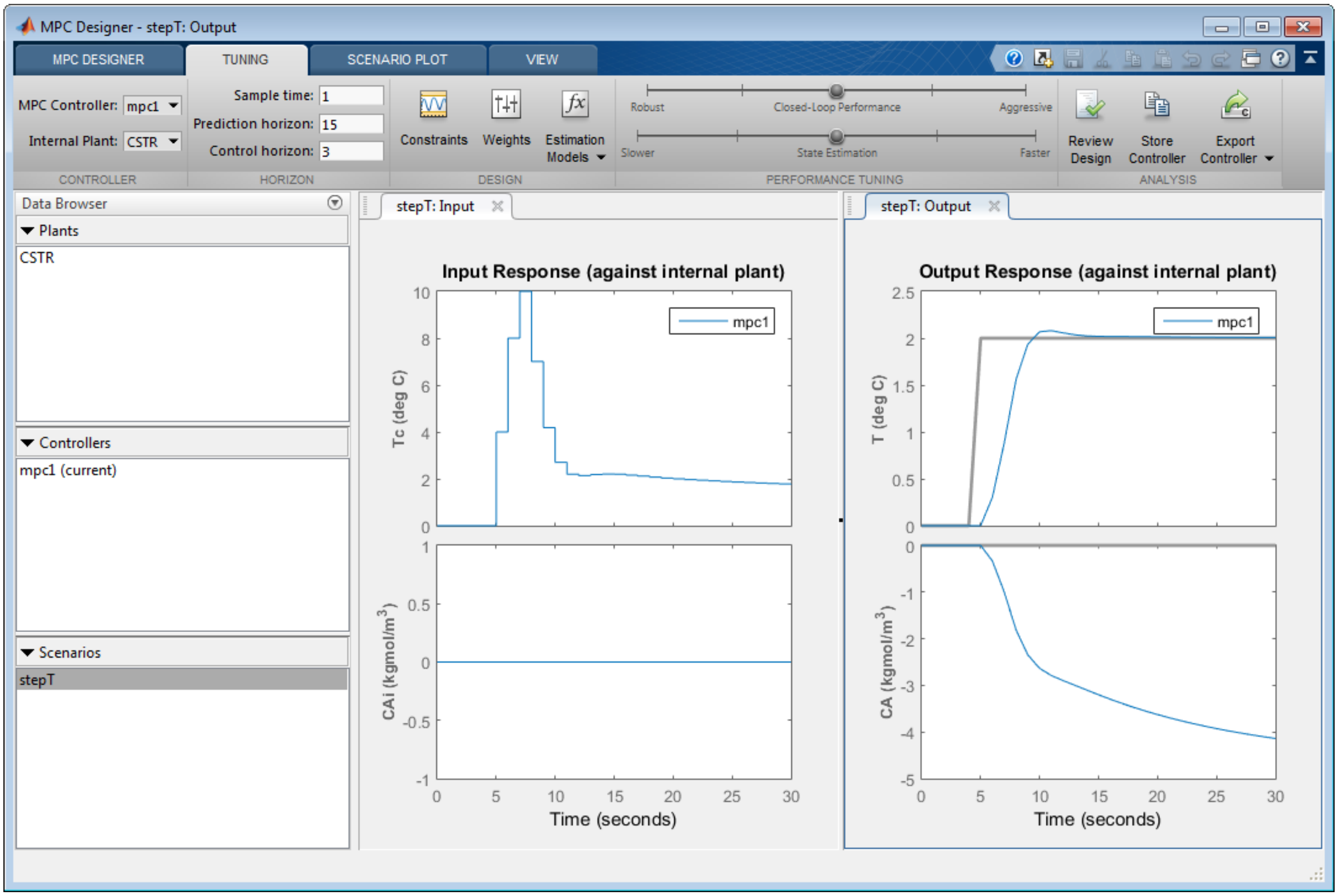
Constraints (mpc1)

Input Constraints

Channel	Type	Min	Max	RateMin	RateMax
u(1)	MV	-10	10	-4	4

+ Constraint Softening Settings

Click **OK**.



MPC Designer - stepT: Output

MPC Controller: mpc1

Internal Plant: CSTR

Sample time: 1

Prediction horizon: 15

Control horizon: 3

The **Input Response** plot shows the constrained manipulated variable control actions. Even with the constrained rate of change, the coolant temperature rises quickly to its maximum limit within three control intervals.

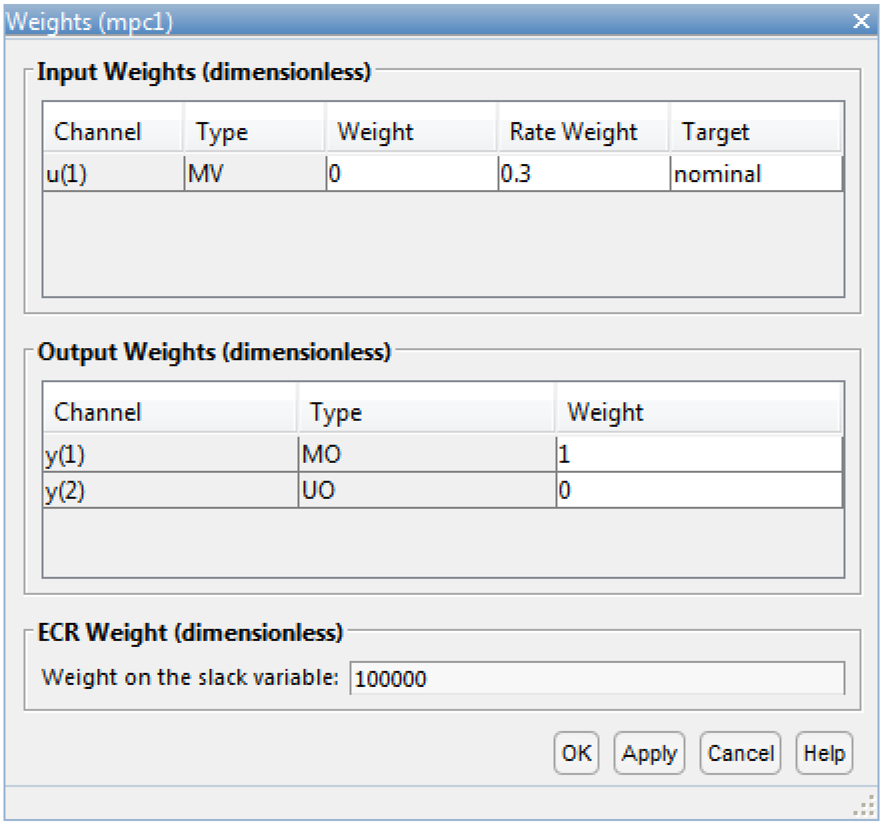
Specify Controller Tuning Weights

On the **Tuning** tab, in the **Design** section, click **Weights**.

In the **Input Weights** table, increase the manipulated variable (MV) **Rate Weight** to `0.3`. Increasing the MV rate weight penalizes large MV changes in the controller optimization cost function.

In the **Output Weights** table, keep the default **Weight** values. By default, all unmeasured outputs have zero weights.

Since there is only one manipulated variable, if the controller tries to hold both outputs at specific setpoints, one or both outputs will exhibit steady-state error in their responses. Since the controller ignores setpoints for outputs with zero weight, setting the concentration output weight to zero allows reactor temperature setpoint tracking with zero steady-state error.



Input Weights (dimensionless)

Channel	Type	Weight	Rate Weight	Target
u(1)	MV	0	0.3	nominal

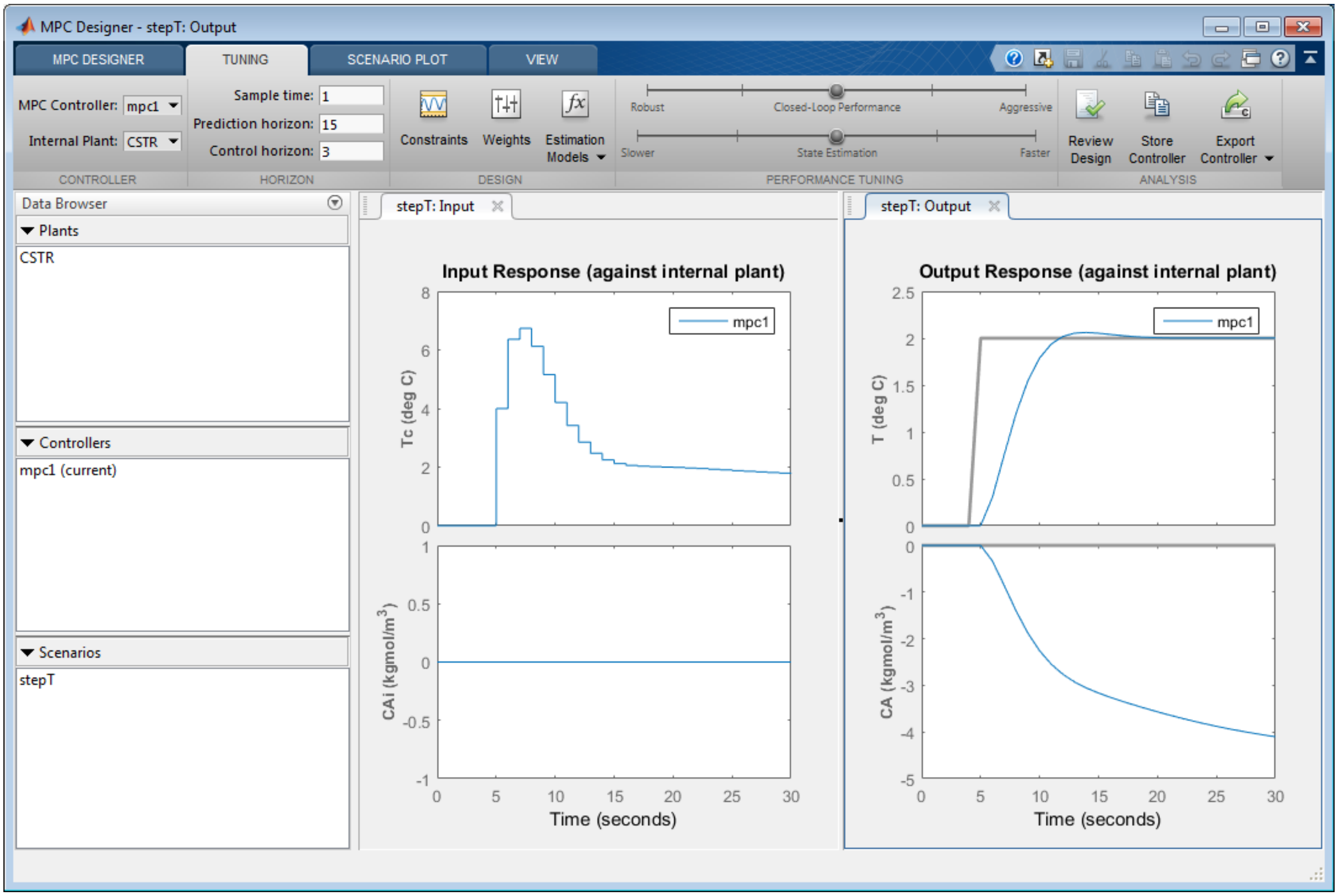
Output Weights (dimensionless)

Channel	Type	Weight
y(1)	MO	1
y(2)	UO	0

ECR Weight (dimensionless)

Weight on the slack variable: 100000

Click **OK**.


MPC Controller: mpc1

Internal Plant: CSTR

Sample time: 1

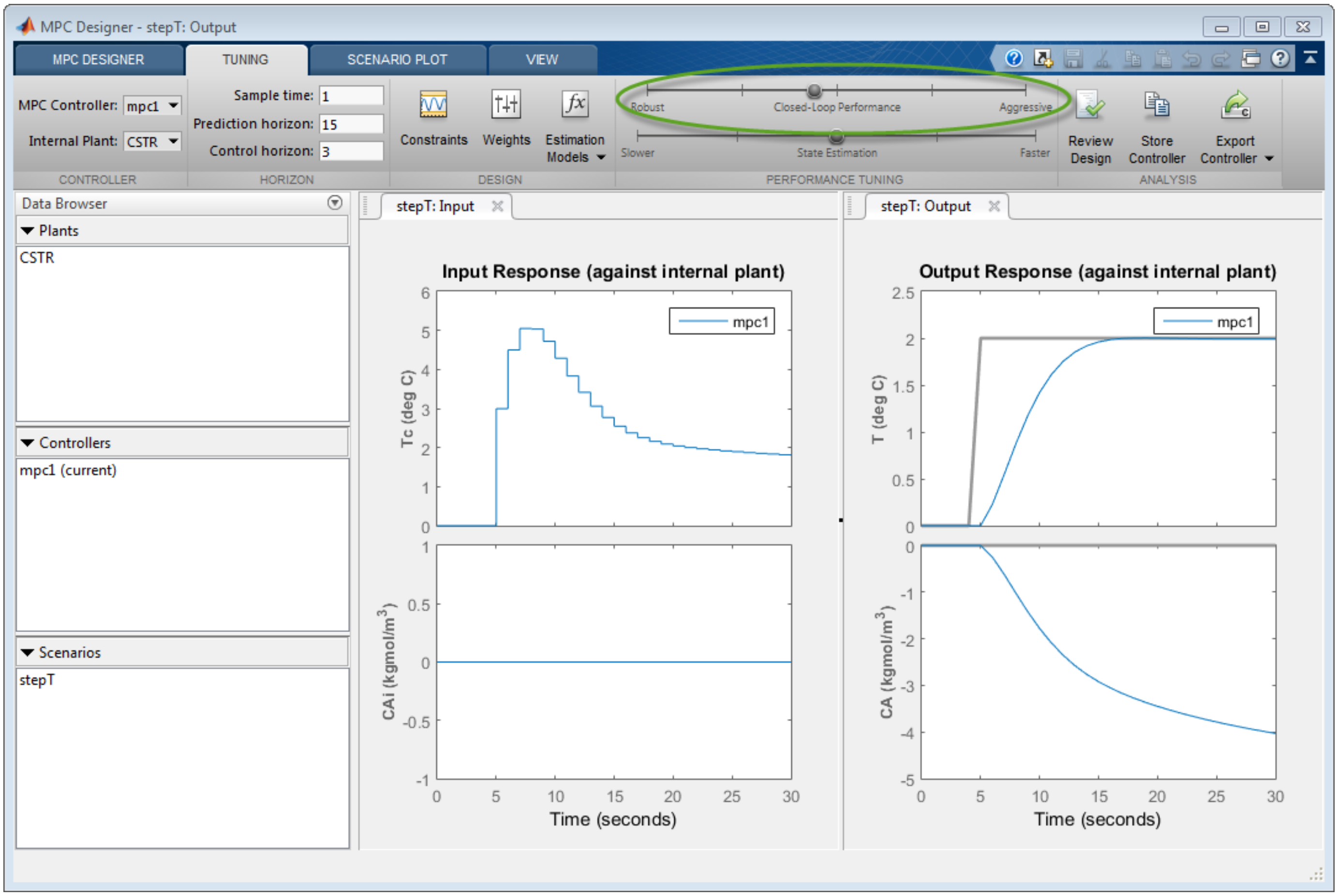
Prediction horizon: 15

Control horizon: 3

The **Input Response** plot shows the more conservative control actions, which result in a slower **Output Response**.

Eliminate Output Overshoot

Suppose the application demands zero overshoot in the output response. On the **Performance Tuning** tab, drag the **Closed-Loop Performance** slider to the left until the **Output Response** has no overshoot. Moving this slider to the left simultaneously increases the manipulated variable rate weight of the controller and decreases the output variable weight, producing a more robust controller.



MPC Designer - stepT: Output

MPC Controller: mpc1

Internal Plant: CSTR

Sample time: 1

Prediction horizon: 15

Control horizon: 3

When you adjust the controller tuning weights using the **Closed-Loop Performance** slider, **MPC Designer** does not change the weights you specified in the Weights dialog box. Instead, the slider controls an adjustment factor, which is used with the user-specified weights to define the actual controller weights.

This factor is `1` when the slider is centered; its value decreases as the slider moves left and increases as the slider moves right. The weighting factor multiplies the manipulated variable and output variable weights and divides the manipulated variable rate weights from the Weights dialog box.

To view the actual controller weights, export the controller to the MATLAB workspace, and view the `Weights` property of the exported controller object.

Test Controller Disturbance Rejection

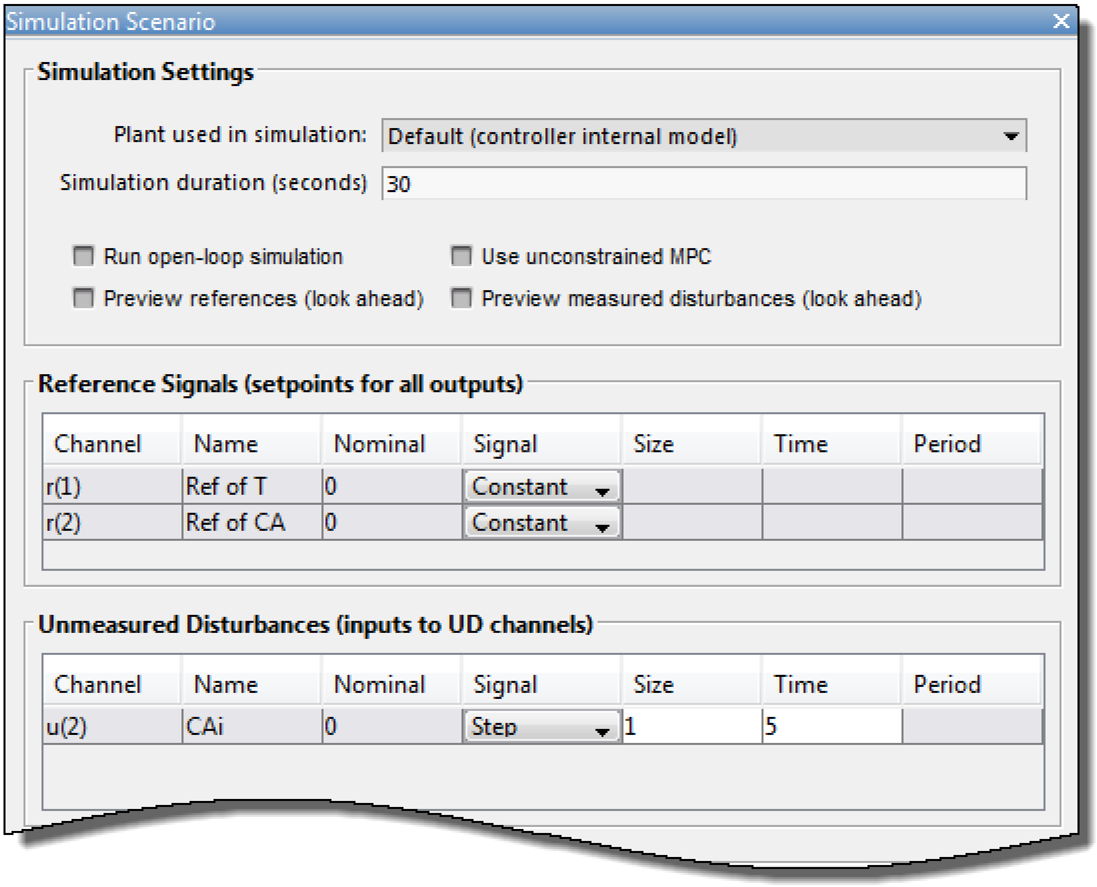
In a process control application, disturbance rejection is often more important than setpoint tracking. Simulate the controller response to a step change in the feed concentration unmeasured disturbance.

On the **MPC Designer** tab, in the **Scenario** section, click **Plot Scenario** > **New Scenario**.

In the Simulation Scenario dialog box, set the **Simulation duration** to 30 seconds.

In the **Unmeasured Disturbances** table, in the **Signal** drop-down list, select `Step`.

In the **Time** column, specify a step time of 5 seconds.



Simulation Scenario

Simulation Settings

Plant used in simulation: Default (controller internal model)

Simulation duration (seconds) 30

□ Run open-loop simulation □ Use unconstrained MPC

□ Preview references (look ahead) □ Preview measured disturbances (look ahead)

Reference Signals (setpoints for all outputs)

Channel	Name	Nominal	Signal	Size	Time	Period
r(1)	Ref of T	0	Constant			
r(2)	Ref of CA	0	Constant			

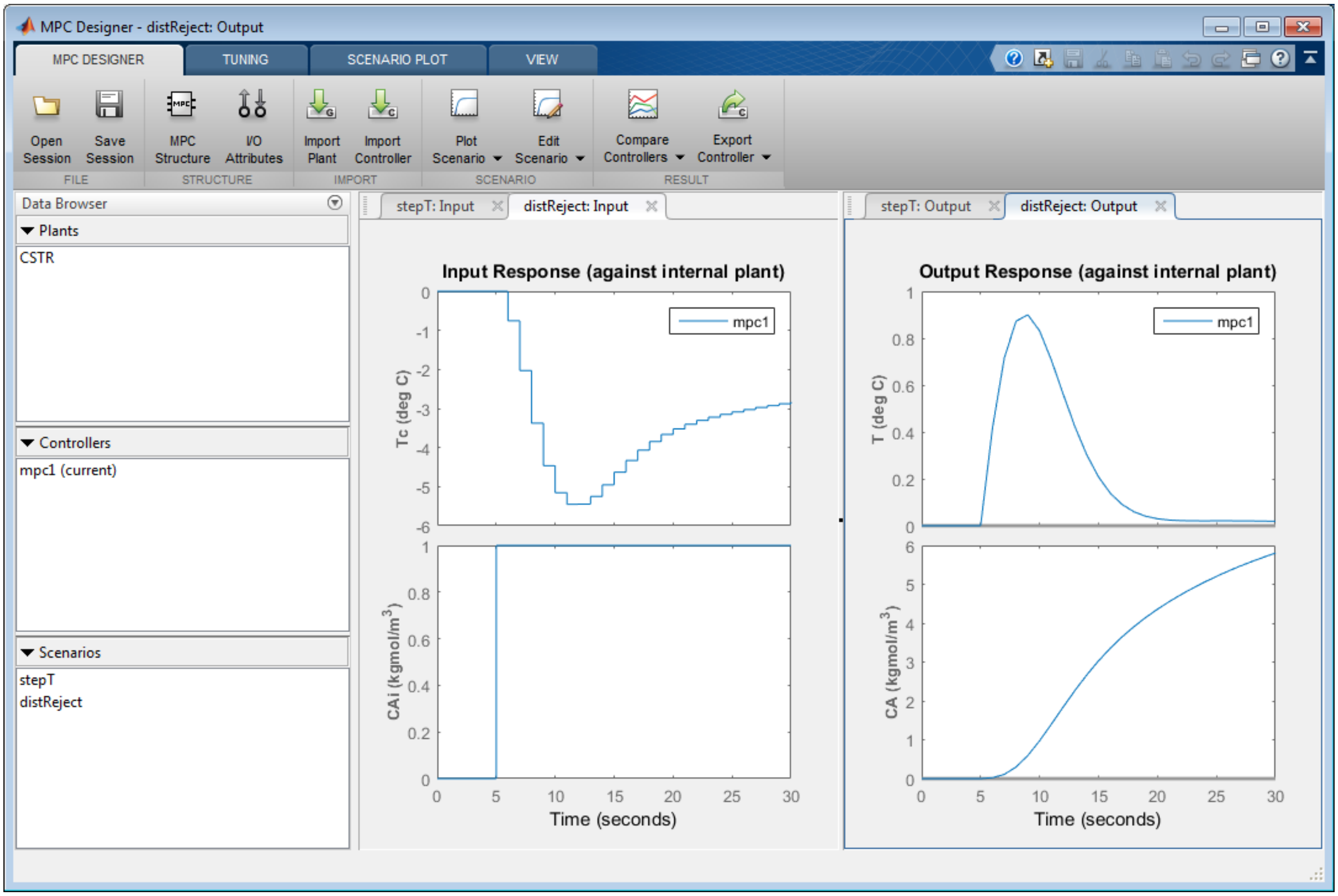
Unmeasured Disturbances (inputs to UD channels)

Channel	Name	Nominal	Signal	Size	Time	Period
u(2)	CAi	0	Step	1	5	

Click **OK**.

The app adds new scenario to the **Data Browser** and creates new corresponding **Input Response** and **Output Response** plots.

In the **Data Browser**, in the **Scenarios** section, rename `NewScenario` to `distReject`.



In the **Output Response** plots, the controller returns the reactor temperature, **T**, to a value near its setpoint as expected. However, the required control actions cause an increase in the output concentration, **CA** to 6 kgmol/m^3.

Specify Concentration Output Constraint

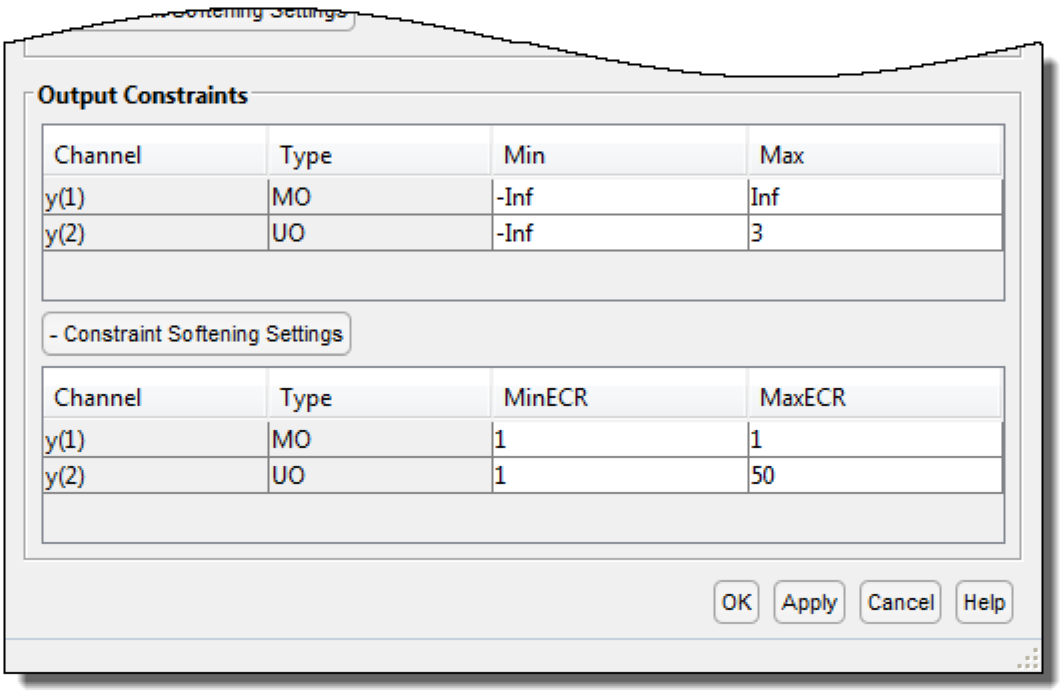
Previously, you defined the controller tuning weights to achieve the primary control objective of tracking the reactor temperature setpoint with zero steady-state error. Doing so enables the unmeasured reactor concentration to vary freely. Suppose that unwanted reactions occur once the reactor concentration exceeds a 3 kgmol/m^3. To limit the reactor concentration, specify an output constraint.

On the **Tuning** tab, in the **Design** section, click **Constraints**.

In the Constraints dialog box, in the **Output Constraints** section, the second row of the table, specify a **Max** unmeasured output (UO) value of 3.

In the **Output Constraints** section, click **Constraint Softening Settings**.

By default, all output constraints are soft, meaning that their **MinECR** and **MaxECR** values are greater than zero. To soften the unmeasured output (UO) constraint further, increase its **MaxECR** value.



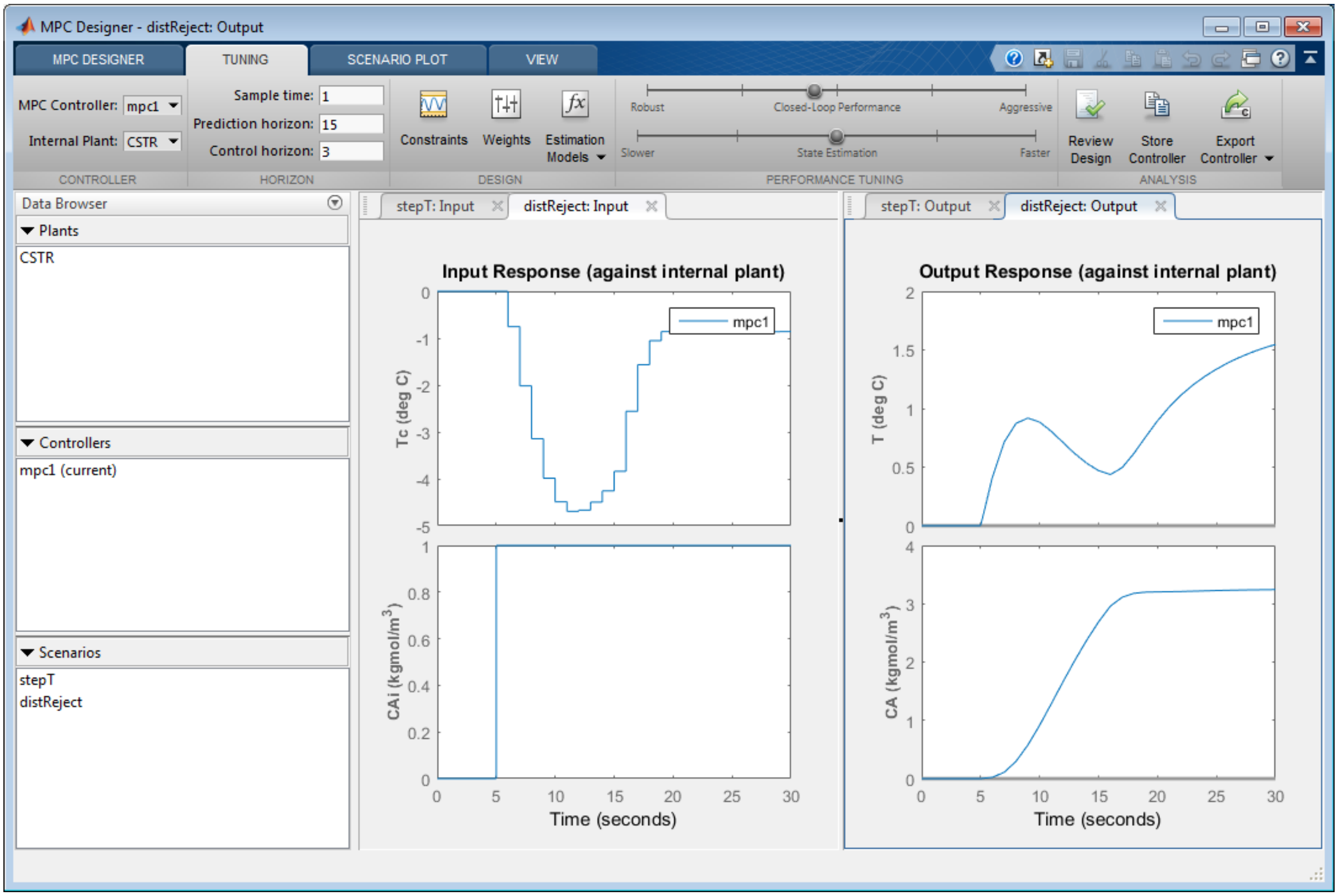
Output Constraints

Channel	Type	Min	Max
y(1)	MO	-Inf	Inf
y(2)	UO	-Inf	3

- Constraint Softening Settings

Channel	Type	MinECR	MaxECR
y(1)	MO	1	1
y(2)	UO	1	50

Click **OK**.



MPC Designer - distReject: Output

MPC Controller: mpc1

Internal Plant: CSTR

Sample time: 1

Prediction horizon: 15

Control horizon: 3

In the **Output Response** plots, once the reactor concentration, **CA**, approaches 3 kgmol/m^3, the reactor temperature, **T**, starts to increase. Since there is only one manipulated variable, the controller makes a compromise between the two competing control objectives: Temperature control and constraint satisfaction. A softer output constraint enables the controller to sacrifice the constraint requirement more to achieve improved temperature tracking.

Since the output constraint is soft, the controller maintain adequate temperature control by allowing a small concentration constraint violation. In general, depending on your application requirements, you can experiment with different constraint settings to achieve an acceptable control objective compromise.

Export Controller

In the **Tuning** tab, in the **Analysis** section, click **Export Controller** to save the tuned controller, `mpc1`, to the MATLAB workspace.

Delete Plants, Controllers, and Scenarios

To delete a plant, controller, or scenario, in the **Data Browser**, right-click the item you want to delete, and select **Delete**. You can also click the item and hit **Delete** on the keyboard.

You cannot delete the current controller. Also, you cannot delete a plant or scenario if it is the only listed plant or scenario.

If a plant is used by any controller or scenario, you cannot delete the plant.

To delete multiple plants, controllers, or scenarios, hold **Shift** and click each item that you want to delete.

References

[1] Seborg, D. E., T. F. Edgar, and D. A. Mellichamp, *Process Dynamics and Control*, 2nd Edition, Wiley, 2004, pp. 34–36 and 94–95.

See Also

MPC Designer

More About

- “Specify Constraints”
- “Tune Weights”
- “Design MPC Controller in Simulink” on page 3-31

Design MPC Controller at the Command Line

This example shows how to create and test a model predictive controller from the command line.

Define Plant Model

This example uses the plant model described in “Design Controller Using MPC Designer” on page 3-2. Create a state-space model of the plant and set some of the optional model properties.

```
A = [-0.0285 -0.0014; -0.0371 -0.1476];
B = [-0.0850 0.0238; 0.0802 0.4462];
C = [0 1; 1 0];
D = zeros(2,2);
CSTR = ss(A,B,C,D);

CSTR.InputName = {'T_c','C_A_i'};
CSTR.OutputName = {'T','C_A'};
CSTR.StateName = {'C_A','T'};
CSTR.InputGroup.MV = 1;
CSTR.InputGroup.UD = 2;
CSTR.OutputGroup.MO = 1;
CSTR.OutputGroup.UO = 2;
```

Create Controller

To improve the clarity of the example, suppress Command Window messages from the MPC controller.

```
old_status = mpcverbosity('off');
```

Create a model predictive controller with a control interval, or sample time, of 1 second, and with all other properties at their default values.

```
Ts = 1;
MPCobj = mpc(CSTR,Ts)
```

```
MPC object (created on 23-Feb-2021 23:52:02):
---------------------------------------------
Sampling time:      1 (seconds)
Prediction Horizon: 10
Control Horizon:    2

Plant Model:
                                      --------------
      1  manipulated variable(s)   -->|  2 states  |
                                      |            |-->  1 measured output(s)
      0  measured disturbance(s)   -->|  2 inputs  |
                                      |            |-->  1 unmeasured output(s)
      1  unmeasured disturbance(s) -->|  2 outputs |
                                      --------------
Indices:
  (input vector)    Manipulated variables: [1 ]
                  Unmeasured disturbances: [2 ]
  (output vector)        Measured outputs: [1 ]
                       Unmeasured outputs: [2 ]

Disturbance and Noise Models:
        Output disturbance model: default (type "getoutdist(MPCobj)" for details)
         Input disturbance model: default (type "getindist(MPCobj)" for details)
         Measurement noise model: default (unity gain after scaling)

Weights:
        ManipulatedVariables: 0
    ManipulatedVariablesRate: 0.1000
             OutputVariables: [1 0]
                         ECR: 100000

State Estimation:  Default Kalman Filter (type "getEstimator(MPCobj)" for details)

Unconstrained
```

View and Modify Controller Properties

Display a list of the controller properties and their current values.

```
get(MPCobj)
```

```
                         Ts: 1
       PredictionHorizon (P): 10
          ControlHorizon (C): 2
                       Model: [1x1 struct]
   ManipulatedVariables (MV): [1x1 struct]
        OutputVariables (OV): [1x2 struct]
   DisturbanceVariables (DV): [1x1 struct]
                 Weights (W): [1x1 struct]
                   Optimizer: [1x1 struct]
                       Notes: {}
                    UserData: []
                     History: 23-Feb-2020 23:52:02
```

The displayed `History` value will be different for your controller, since it depends on when the controller was created. For a description of the editable properties of an MPC controller, enter `mpcprops` at the command line.

Use dot notation to modify these properties. For example, change the prediction horizon to `15`.

```
MPCobj.PredictionHorizon = 15;
```

You can abbreviate property names provided that the abbreviation is unambiguous.

Many of the controller properties are structures containing additional fields. Use dot notation to view and modify these field values. For example, you can set the measurement units for the controller output variables. The `OutputUnit` property is for display purposes only and is optional.

```
MPCobj.Model.Plant.OutputUnit = {'Deg C','kmol/m^3'};
```

By default, the controller has no constraints on manipulated variables and output variables. You can view and modify these constraints using dot notation. For example, set constraints for the controller manipulated variable.

```
MPCobj.MV.Min = -10;
MPCobj.MV.Max = 10;
MPCobj.MV.RateMin = -3;
MPCobj.MV.RateMax = 3;
```

You can also view and modify the controller tuning weights. For example, modify the weights for the manipulated variable rate and the output variables.

```
MPCobj.W.ManipulatedVariablesRate = 0.3;
MPCobj.W.OutputVariables = [1 0];
```

You can also define time-varying constraints and weights over the prediction horizon, which shifts at each time step. Time-varying constraints have a nonlinear effect when they are active. For example, to force the manipulated variable to change more slowly towards the end of the prediction horizon, enter:

```
MPCobj.MV.RateMin = [-4; -3.5; -3; -2.5];
```

```
MPCobj.MV.RateMax = [4; 3.5; 3; 2.5];
```

The `-2.5` and `2.5` values are used for the fourth step and beyond.

Similarly, you can specify different output variable weights for each step of the prediction horizon. For example, enter:

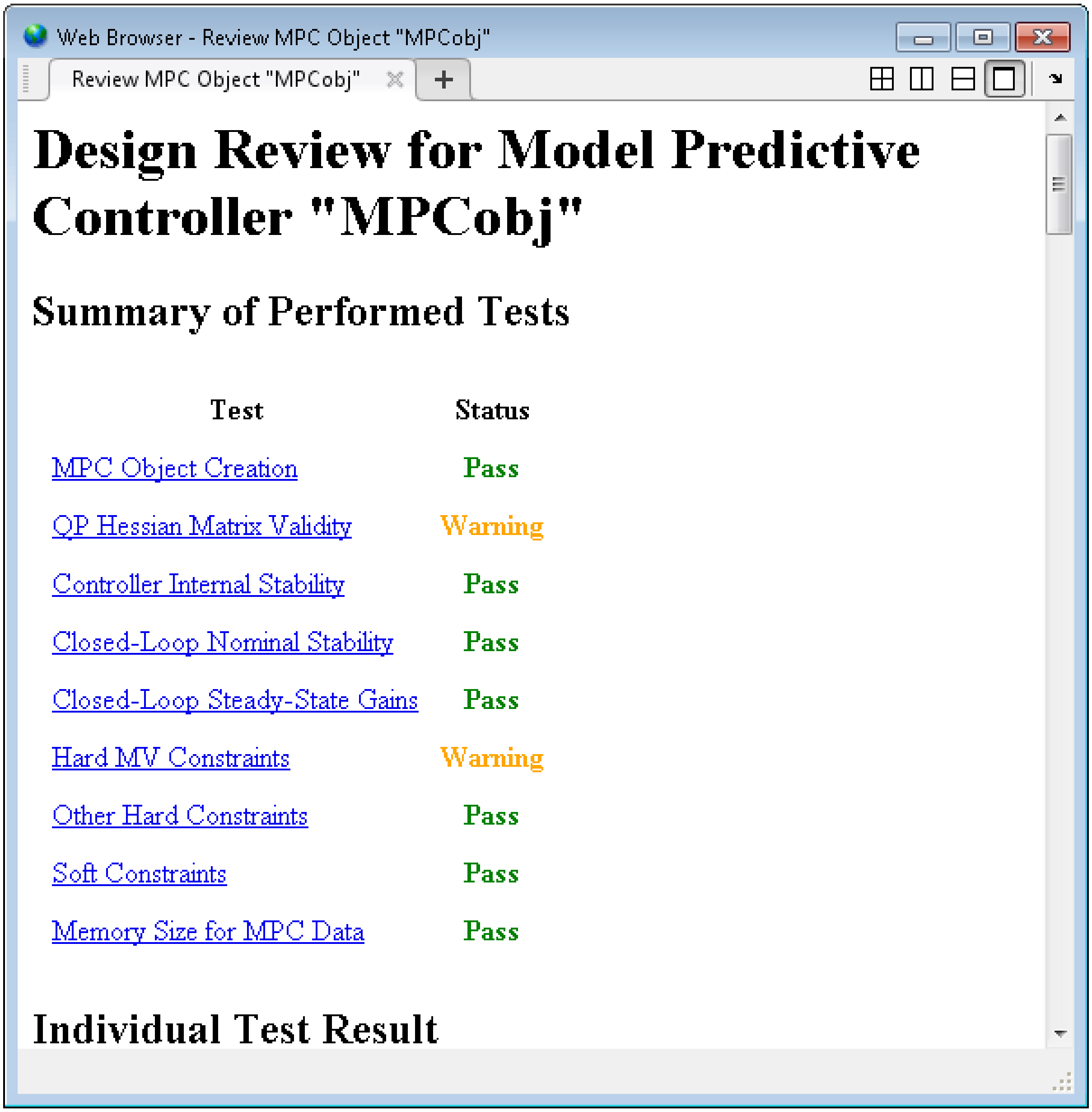
```
MPCobj.W.OutputVariables = [0.1 0; 0.2 0; 0.5 0; 1 0];
```

You can also modify the disturbance rejection characteristics of the controller. See `setEstimator`, `setindist`, and `setoutdist` for more information.

Review Controller Design

Generate a report on potential run-time stability and performance issues.

```
review(MPCobj)
```


Web Browser - Review MPC Object "MPCobj"
Review MPC Object "MPCobj"

Design Review for Model Predictive Controller "MPCobj"

Summary of Performed Tests

Test	Status
MPC Object Creation	Pass
QP Hessian Matrix Validity	Warning
Controller Internal Stability	Pass
Closed-Loop Nominal Stability	Pass
Closed-Loop Steady-State Gains	Pass
Hard MV Constraints	Warning
Other Hard Constraints	Pass
Soft Constraints	Pass
Memory Size for MPC Data	Pass

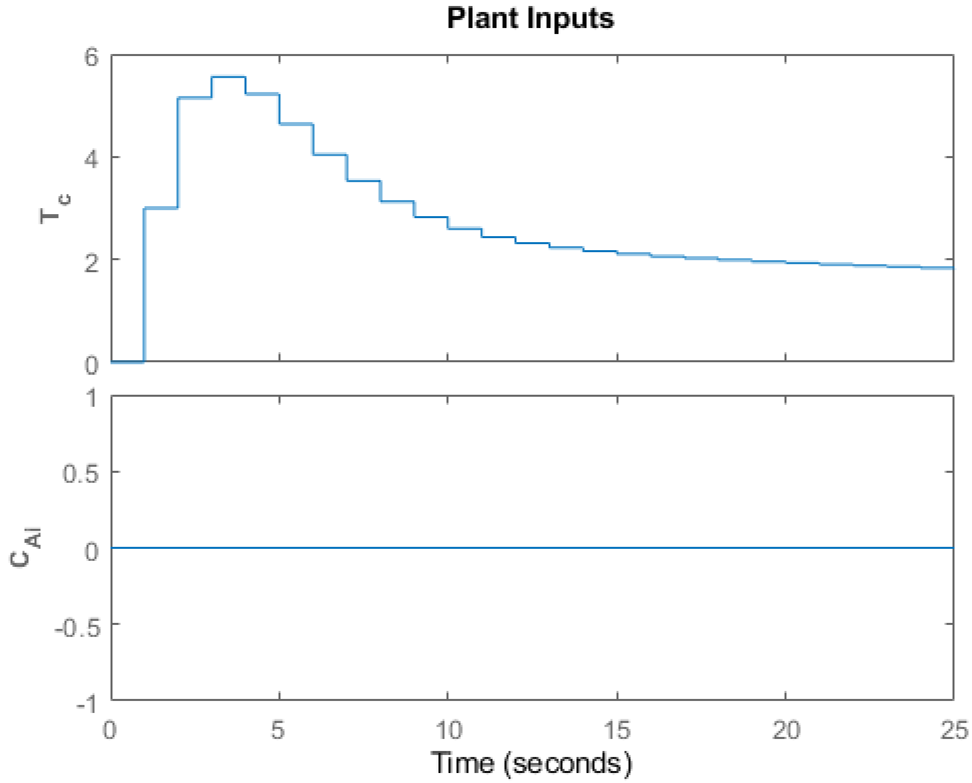
Individual Test Result

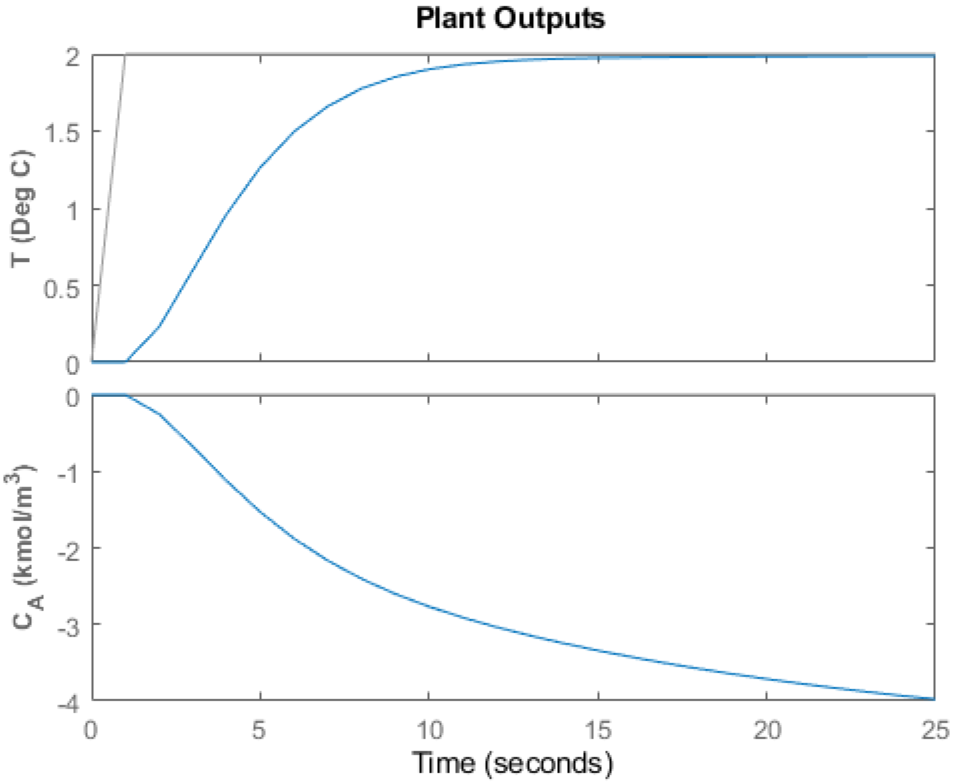
In this example, the `review` command found two potential issues with the design. The first warning asks whether the user intends to have a weight of zero on the `C_A` output. The second warning advises the user to avoid having hard constraints on both `MV` and `MVRate`.

Perform Linear Simulations

Use the `sim` function to run a linear simulation of the system. For example, simulate the closed-loop response of `MPCobj` for `26` control intervals. Specify setpoints of `2` and `0` for the reactor temperature and the residual concentration respectively. The setpoint for the residual concentration is ignored because the tuning weight for the second output is zero.

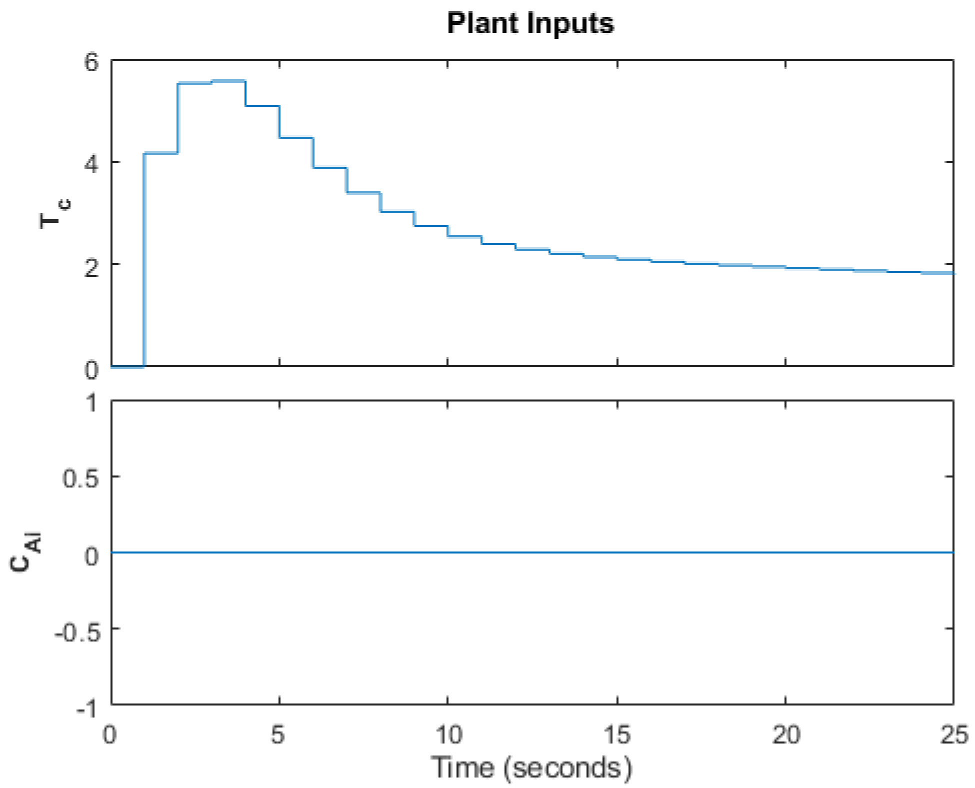
```
T = 26;
r = [0 0; 2 0];
sim(MPCobj,T,r)
```

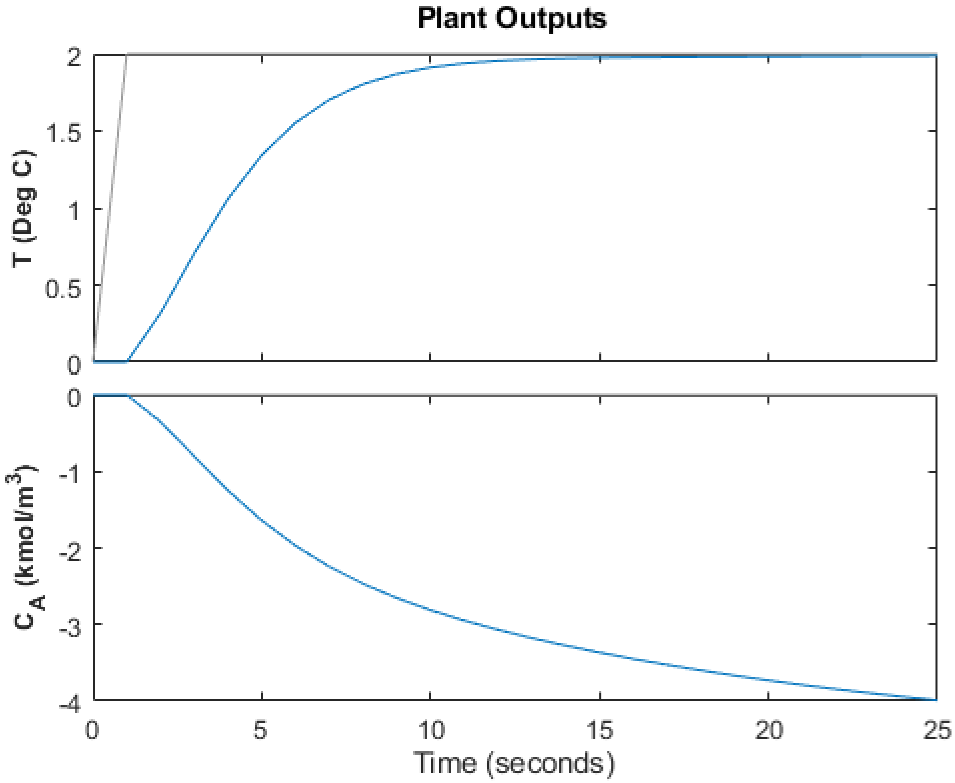

Plant Inputs
6
4
2
0
T c
1
0.5
0
-0.5
-1
C AI
0
5
10
15
20
25
Time (seconds)



You can modify the simulation options using `mpcsimopt`. For example, run a simulation with the manipulated variable constraints turned off.

```
MPCopts = mpcsimopt;
MPCopts.Constraints = 'off';
sim(MPCobj,T,r,MPCopts)
```

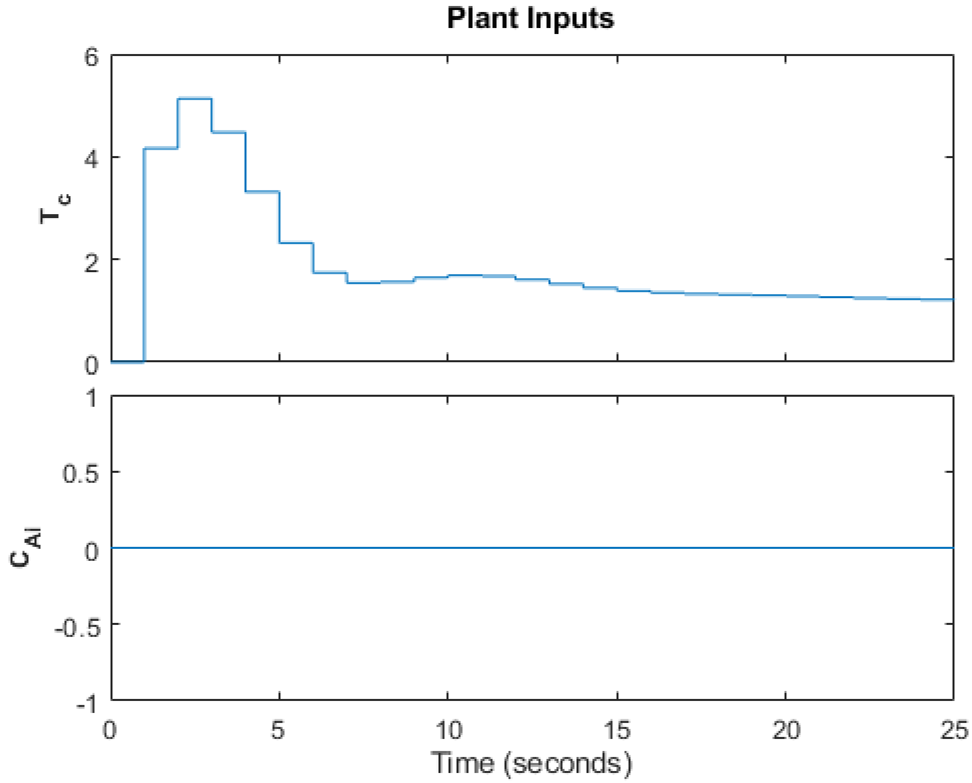

Plant Inputs
T c
6
4
2
0
C AI
1
0.5
0
-0.5
-1
0
5
10
15
20
25
Time (seconds)

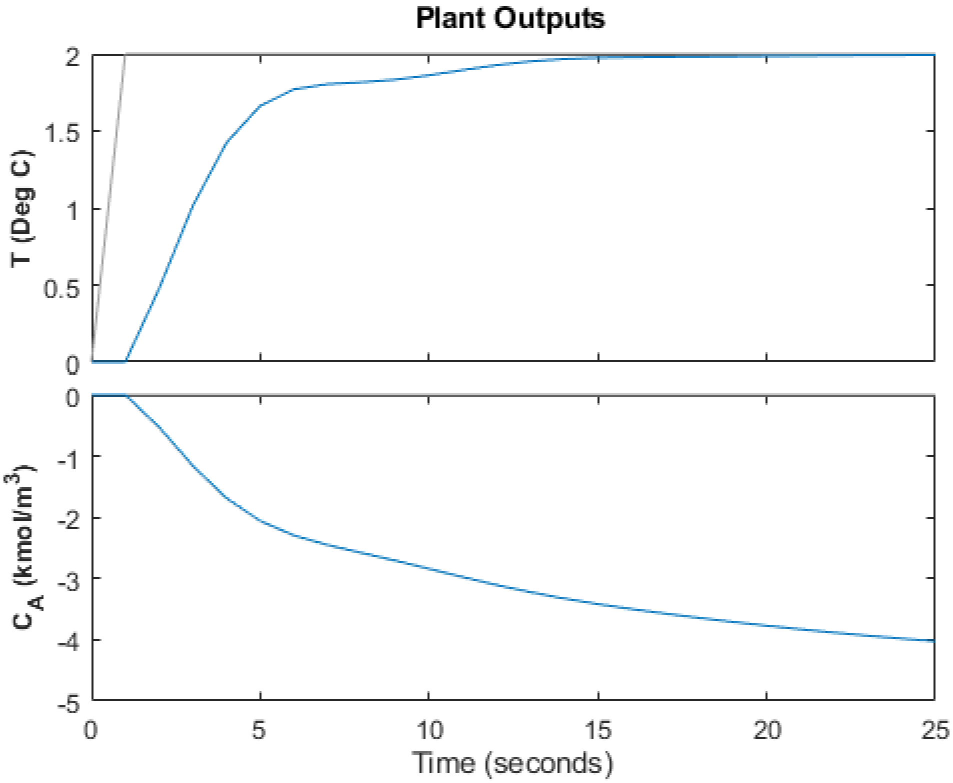


The first move of the manipulated variable now exceeds the specified 3-unit rate constraint.

You can also perform a simulation with a plant/model mismatch. For example, define a plant with 50% larger gains than those in the model used by the controller.

```
Plant = 1.5*CSTR;
MPCopts.Model = Plant;
sim(MPCobj,T,r,MPCopts)
```


Plant Inputs
T c
6
4
2
0
C AI
1
0.5
0
-0.5
-1
0
5
10
15
20
25
Time (seconds)



The plant/model mismatch degrades controller performance slightly. Degradation can be severe and must be tested on a case-by-case basis.

Other options include the addition of a specified noise sequence to the manipulated variables or measured outputs, open-loop simulations, and a look-ahead option for better setpoint tracking or measured disturbance rejection.

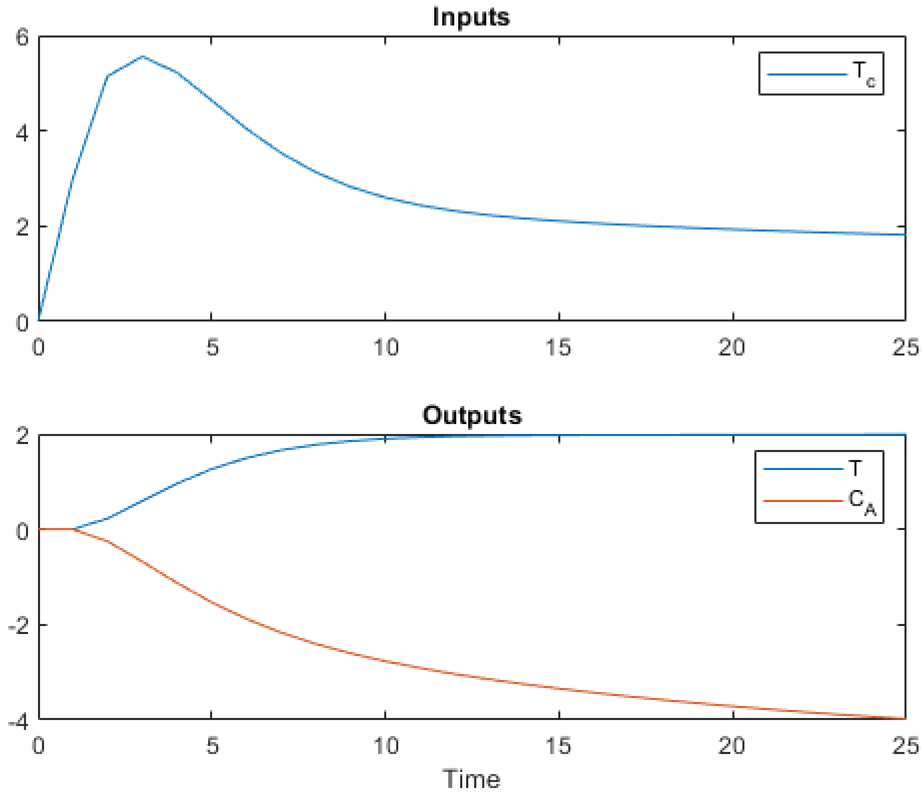
Store Simulation Results

Store the simulation results in the MATLAB Workspace.

```
[y,t,u] = sim(MPCobj,T,r);
```

The syntax suppresses automatic plotting and returns the simulation results. You can use the results for other tasks, including custom plotting. For example, plot the manipulated variable and both output variables in the same figure.

```
figure
subplot(2,1,1)
plot(t,u)
title('Inputs')
legend('T_c')
subplot(2,1,2)
plot(t,y)
title('Outputs')
legend('T','C_A')
xlabel('Time')
```



Restore the `mpcverbosity` setting.

```
mpcverbosity(old_status);
```

See Also

`mpc` | `review` | `sim`

More About

- “MPC Modeling” on page 2-2
- “Design Controller Using MPC Designer” on page 3-2
- “Design MPC Controller in Simulink” on page 3-31

Design MPC Controller in Simulink

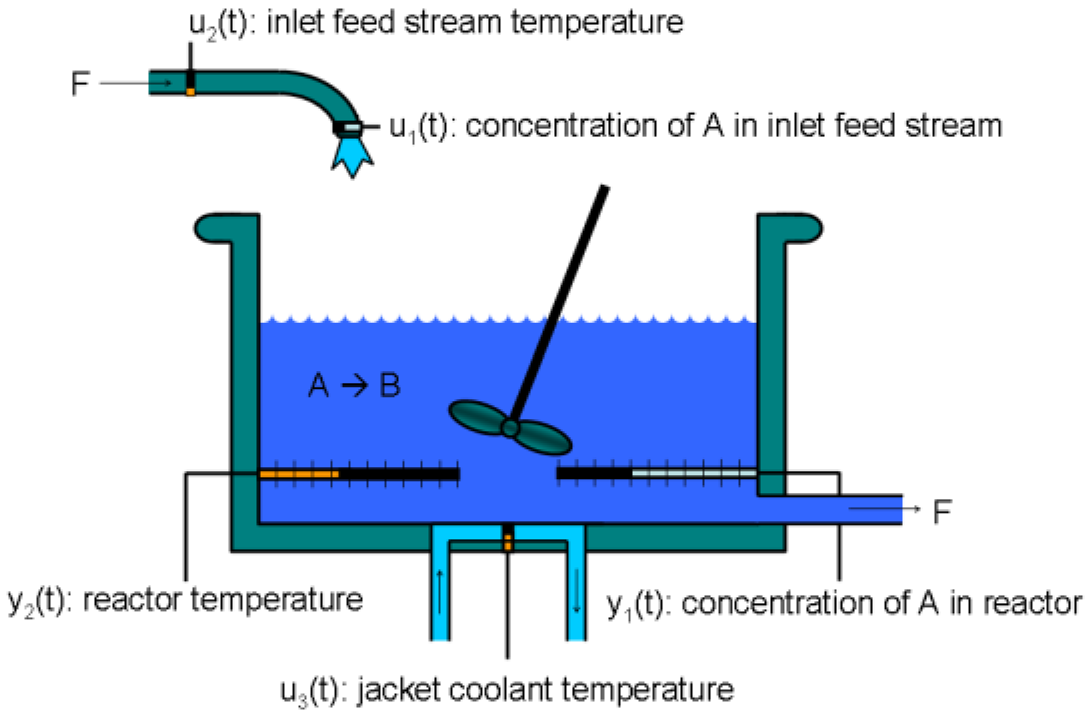
This example shows how to design a model predictive controller for a continuous stirred-tank reactor (CSTR) in Simulink using **MPC Designer**.

This example requires Simulink Control Design software to define the MPC structure by linearizing a nonlinear Simulink model.

If you do not have Simulink Control Design software, you must first create an `mpc` object in the MATLAB workspace. For more information, see “Design Controller Using MPC Designer” on page 3-2 and “Design MPC Controller at the Command Line” on page 3-20.

CSTR Model

A CSTR is a jacketed nonadiabatic tank reactor commonly used in the process industry.


$u_2(t)$: inlet feed stream temperature
F
$u_1(t)$: concentration of A in inlet feed stream
A → B
F
$y_2(t)$: reactor temperature
$y_1(t)$: concentration of A in reactor
$u_3(t)$: jacket coolant temperature

An inlet stream of reagent *A* feeds into the tank at a constant rate. A first-order, irreversible, exothermic reaction takes place to produce the product stream, which exits the reactor at the same rate as the input stream.

The CSTR model has three inputs:

- Feed Concentration (*CAi*) — The concentration of reagent *A* in the feed stream (kgmol/m^3)
- Feed Temperature (*Ti*) — Feed stream temperature (K)
- Coolant Temperature (*Tc*) — Reactor coolant temperature (K)

The two model outputs are:

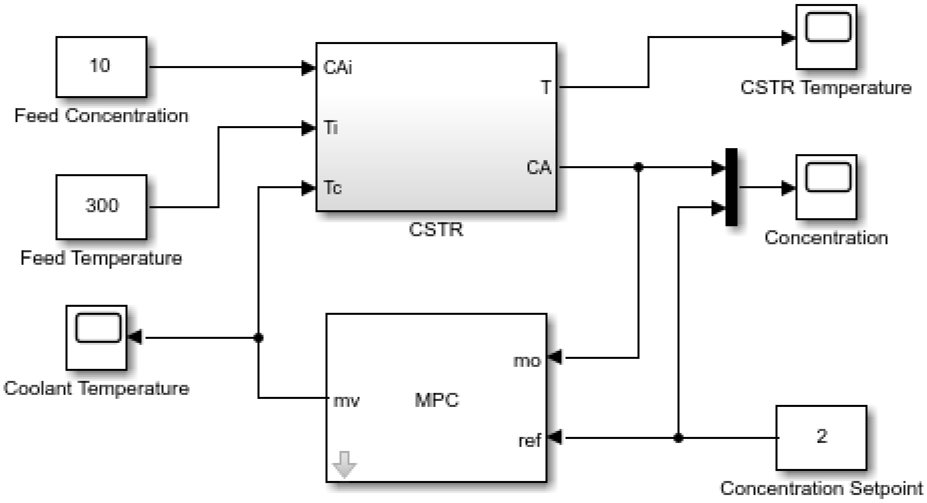
- CSTR Temperature (*T*) — Reactor temperature (K)
- Concentration (*CA*) — Concentration of reagent *A* in the product stream, also referred to as the residual concentration (kgmol/m^3)

The control objective is to maintain the residual concentration, *CA*, at its nominal setpoint by adjusting the coolant temperature, *Tc*. Changes in the feed concentration, *CAi*, and feed temperature, *Ti*, cause disturbances in the CSTR reaction.

The reactor temperature, *T*, is usually controlled. However, for this example, ignore the reactor temperature, and assume that the residual concentration is measured directly.

Open the Simulink model.

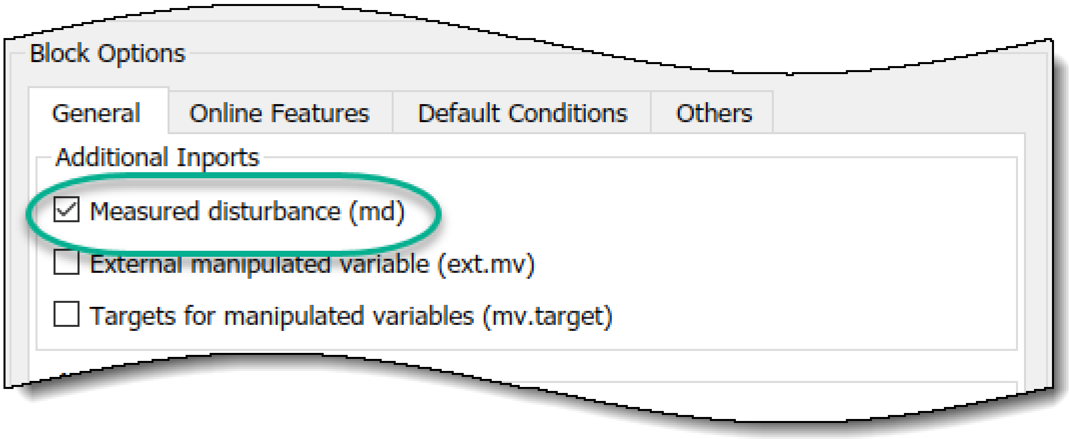
```
open_system('CSTR_ClosedLoop')
```



Connect Measured Disturbance Signal

In the Simulink model window, double-click the MPC Controller block.

In the Block Parameters dialog box, on the **General** tab, select the **Measured disturbance (md)** option.



Block Options

General | Online Features | Default Conditions | Others

Additional Inports

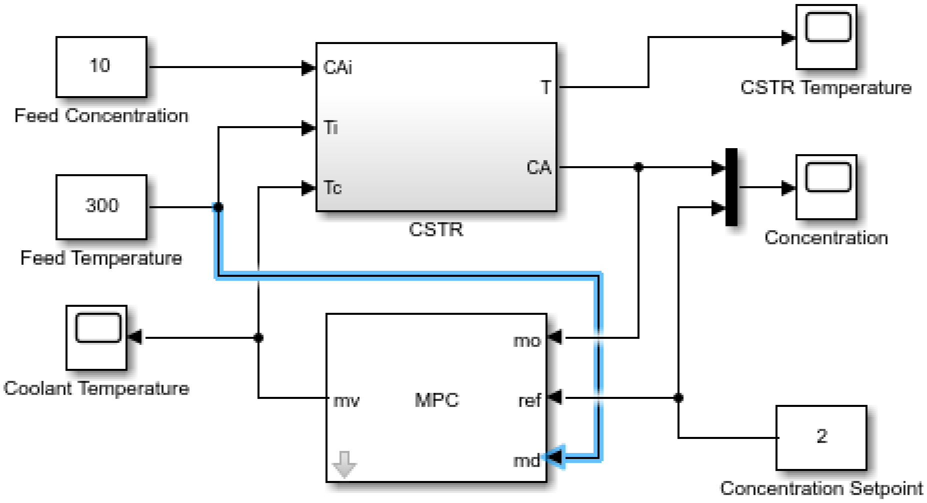
☑ Measured disturbance (md)

☐ External manipulated variable (ext.mv)

☐ Targets for manipulated variables (mv.target)

Click **Apply** to add the `md` input port to the controller block.

In the Simulink model window, connect the Feed Temperature block output to the `md` input port.



Linearize Simulink Model

In this example, you linearize the Simulink model from within **MPC Designer**, which requires Simulink Control Design software. For more information, see “Linearize Simulink Models Using MPC Designer” on page 2-23.

If you do not have Simulink Control Design software, you must first create an `mpc` object in the MATLAB workspace and specify that controller object in the MPC Controller block.

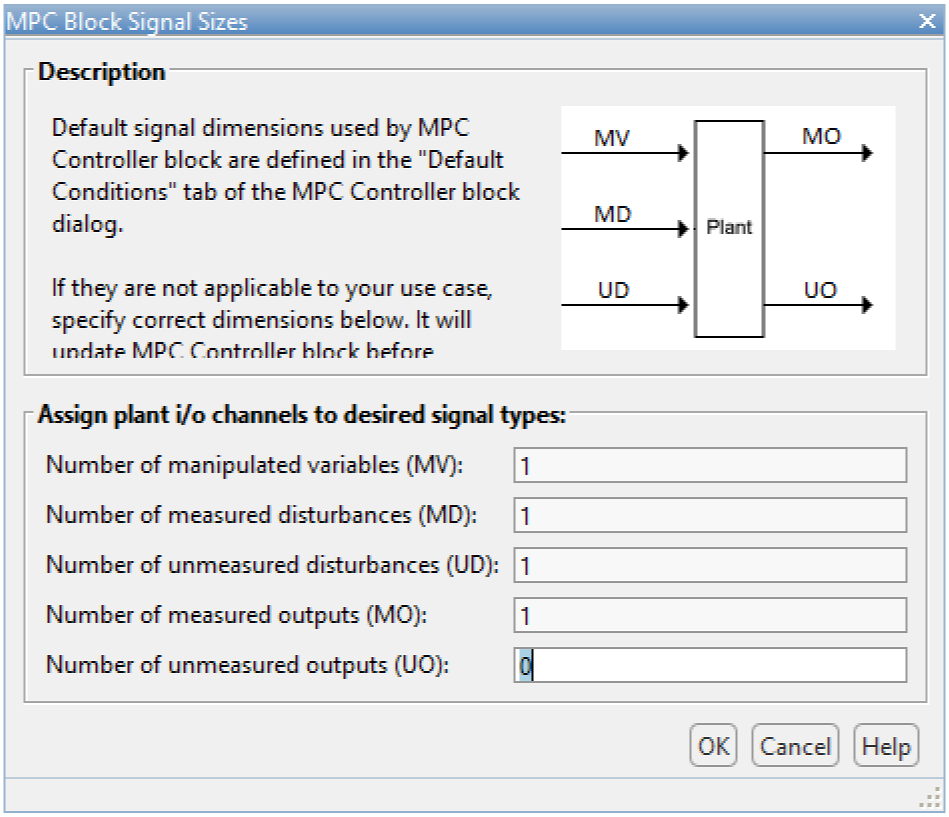
To open **MPC Designer**, open the MPC Controller block and click **Design**.

In **MPC Designer**, on the **MPC Designer** tab, in the **Structure** section, click **MPC Structure**.

In the Define MPC Structure By Linearization dialog box, in the **Controller Sample Time** section, specify a sample time of `0.1`.

In the **MPC Structure** section, click **Change I/O Sizes** to add the unmeasured disturbance and measured disturbance signal dimensions.

In the MPC Block Signal Sizes dialog box, specify the number of input/output channels of each type.



MPC Block Signal Sizes

Description

Default signal dimensions used by MPC Controller block are defined in the "Default Conditions" tab of the MPC Controller block dialog.

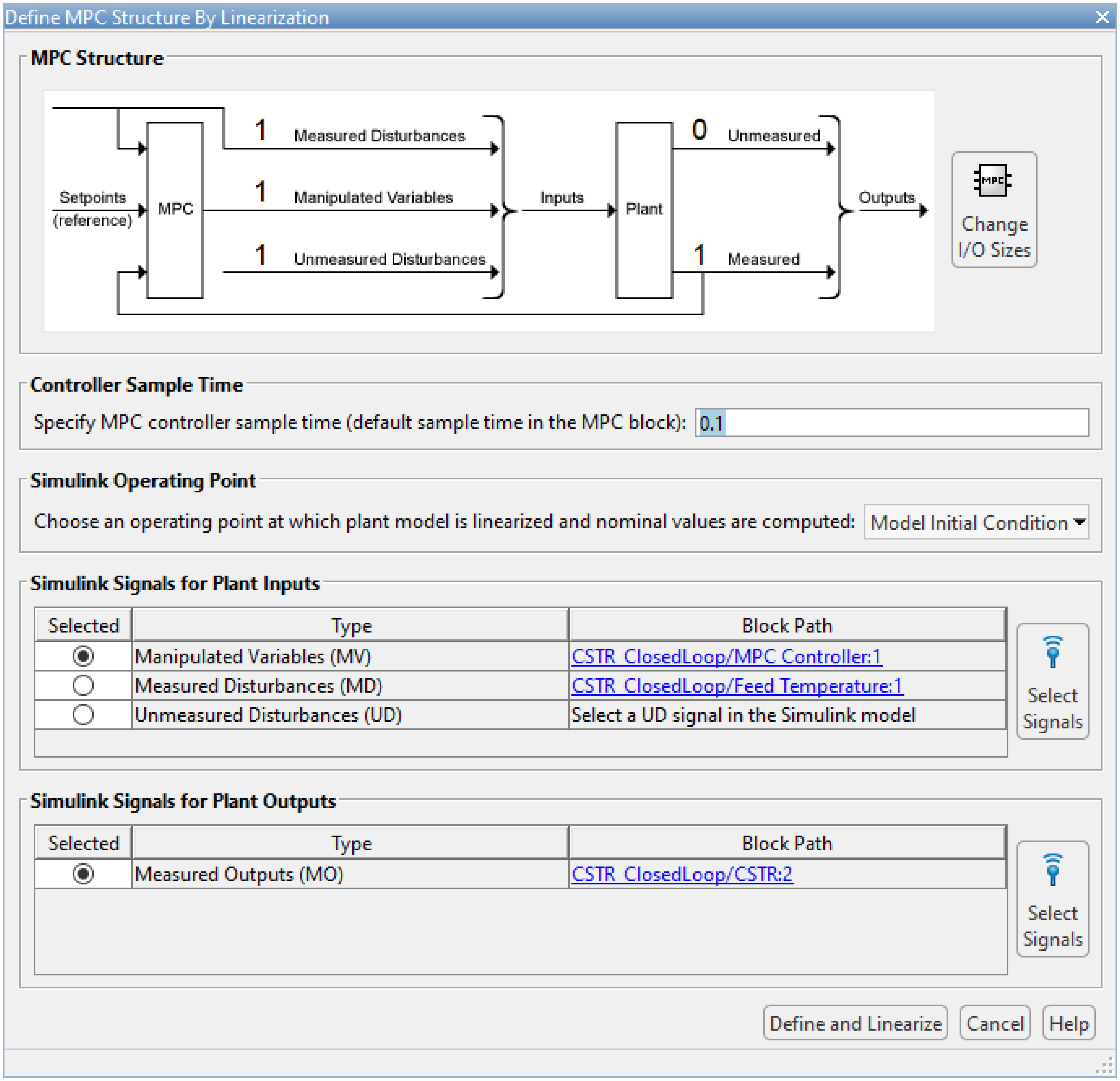
If they are not applicable to your use case, specify correct dimensions below. It will update MPC Controller block before

Assign plant i/o channels to desired signal types:

Number of manipulated variables (MV):	1
Number of measured disturbances (MD):	1
Number of unmeasured disturbances (UD):	1
Number of measured outputs (MO):	1
Number of unmeasured outputs (UO):	0

Click **OK**.

In the Define MPC Structure By Linearization dialog box, in the **Simulink Signals for Plant Inputs** section, the app adds a row for **Unmeasured Disturbances (UD)**.



Controller Sample Time

Specify MPC controller sample time (default sample time in the MPC block): 0.1

Simulink Operating Point

Choose an operating point at which plant model is linearized and nominal values are computed: Model Initial Condition

Simulink Signals for Plant Inputs

Selected	Type	Block Path
●	Manipulated Variables (MV)	CSTR_ClosedLoop/MPC Controller:1
○	Measured Disturbances (MD)	CSTR_ClosedLoop/Feed Temperature:1
○	Unmeasured Disturbances (UD)	Select a UD signal in the Simulink model

Simulink Signals for Plant Outputs

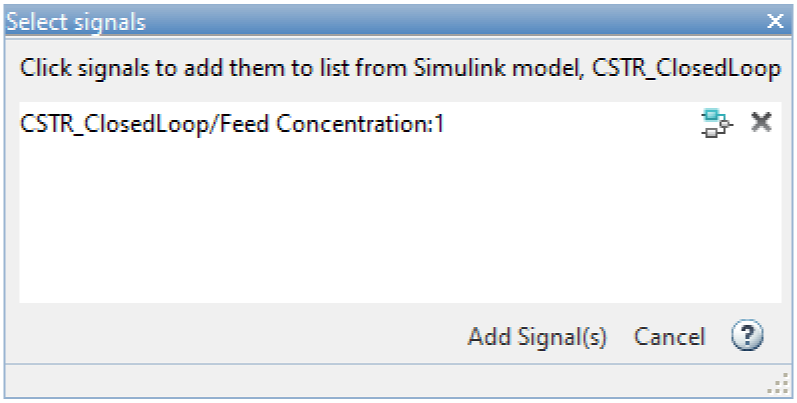
Selected	Type	Block Path
●	Measured Outputs (MO)	CSTR_ClosedLoop/CSTR:2

The manipulated variable, measured disturbance, and measured output are already assigned to their respective Simulink signal lines, which are connected to the MPC Controller block.

In the **Simulink Signals for Plant Inputs** section, select the **Unmeasured Disturbances (UD)** row, and click **Select Signals**.

In the Simulink model window, click the output signal from the Feed Concentration block.

The signal is highlighted and its block path is added to the Select Signal dialog box.



Select signals

Click signals to add them to list from Simulink model, CSTR_ClosedLoop

CSTR_ClosedLoop/Feed Concentration:1

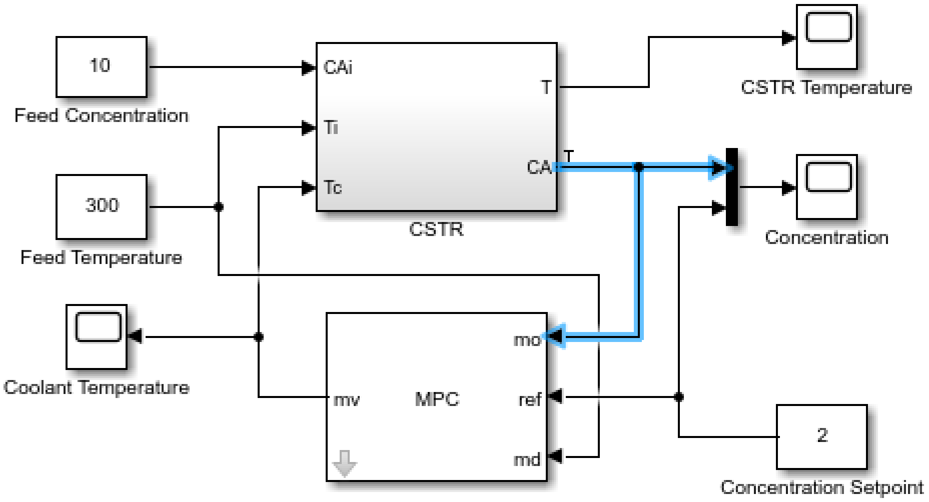
In the Select Signals dialog box, click **Add Signal(s)**.

In the Define MPC Structure By Linearization dialog box, in the **Simulink Signals for Plant Inputs** table, the **Block Path** for the unmeasured disturbance signal is updated.

In this example, you linearize the Simulink model at a steady-state equilibrium operating point where the residual concentration is 2 kgmol/m^3. To compute such an operating point, add the CA signal as a trim output constraint, and specify its target constraint value.

In the Simulink model window, select the signal line connected to CA output port of the CSTR block.

On the **Apps** tab, click **Linearization Manager**. Then, on the **Linearization** tab, in the **Insert Analysis Points** gallery, select **Trim Output Constraint**.

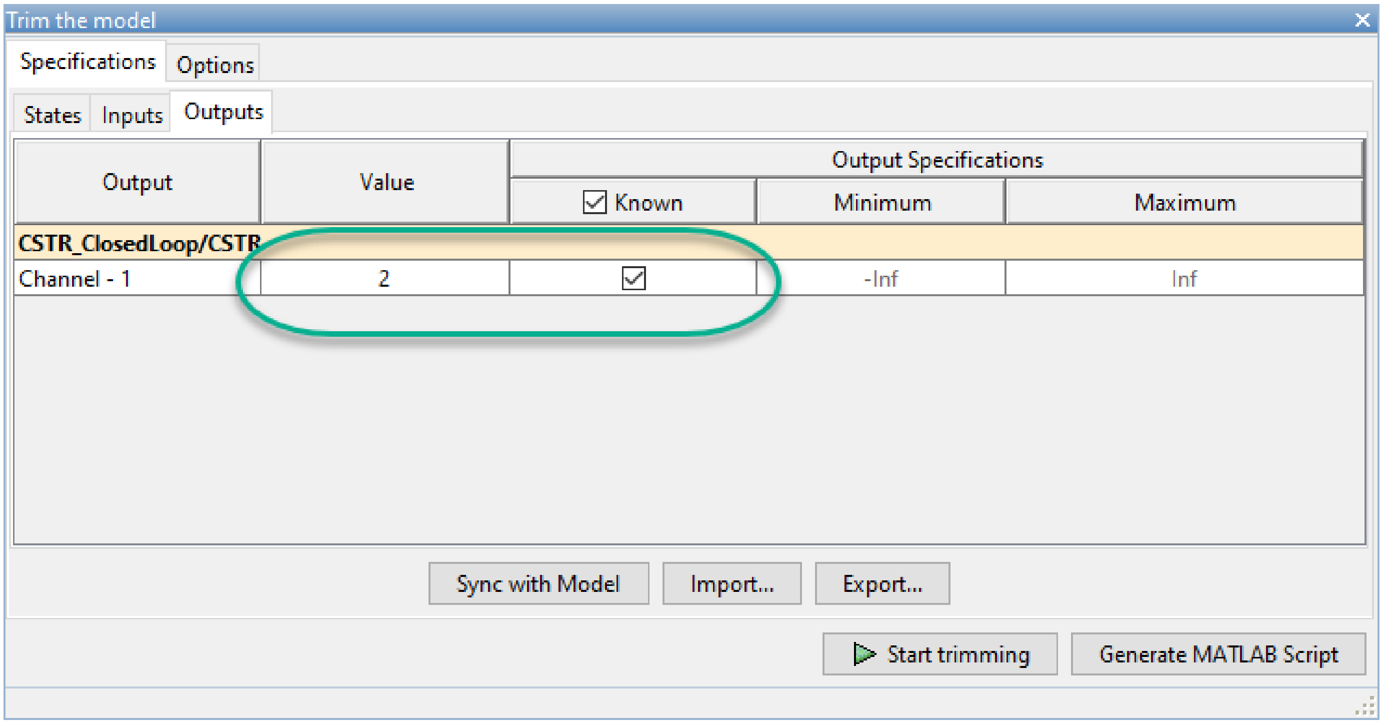


The CA signal can now be used to define output specifications for calculating a model steady-state operating point.

In the Define MPC Structure By Linearization dialog box, in the **Simulink Operating Point** section, in the drop-down list, select **Trim Model**.

When using **MPC Designer** in MATLAB Online, trimming is not supported. You must linearize your model at the model initial conditions.

In the Trim the model dialog box, on the **Outputs** tab, check the box in the **Known** column for `Channel-1` and specify a **Value** of 2.



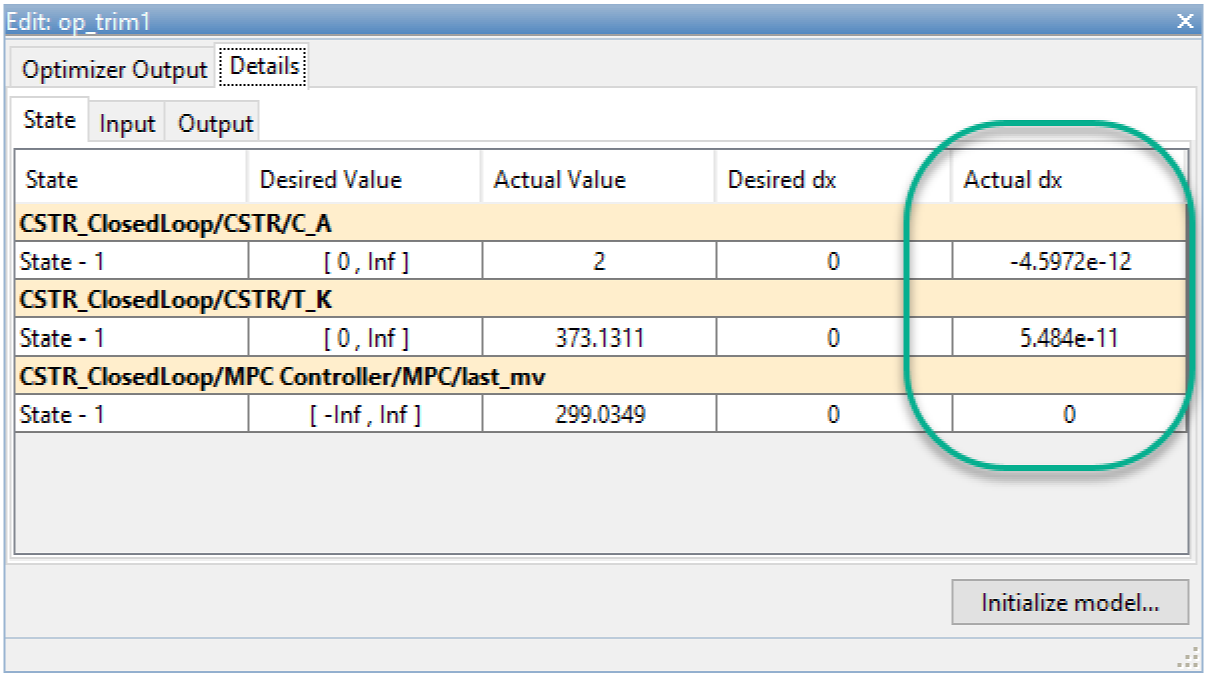
Output	Value	Output Specifications		
		☑ Known	Minimum	Maximum
CSTR_ClosedLoop/CSTR				
Channel - 1	2	☑	-Inf	Inf

This setting constrains the value of the output signal during the operating point search to a known value.

Click **Start Trimming**.

In the Define MPC Structure By Linearization dialog box, in the **Simulink Operating Point** section, the computed operating point, `op_trim1`, is added to the drop-down list and selected.

In the drop-down list, under **View/Edit**, click **Edit op_trim1**.



State	Desired Value	Actual Value	Desired dx	Actual dx
CSTR_ClosedLoop/CSTR/C_A				
State - 1	[0 , Inf]	2	0	-4.5972e-12
CSTR_ClosedLoop/CSTR/T_K				
State - 1	[0 , Inf]	373.1311	0	5.484e-11
CSTR_ClosedLoop/MPC Controller/MPC/last_mv				
State - 1	[-Inf , Inf]	299.0349	0	0

In the Edit dialog box, on the **State** tab, in the **Actual dx** column, the near-zero derivative values indicate that the computed operating point is at steady-state.

To set the initial states of the Simulink model to the operating point values in the **Actual Values** column, click **Initialize model**. Doing so enables you to later simulate the Simulink model at the computed operating point rather than at the default model initial conditions.

In the Initialize Model dialog box, click **OK**.

When setting the model initial conditions, **MPC Designer** exports the operating point to the MATLAB workspace. Also, in the Simulink Configuration Parameters dialog box, in the **Data Import/Export** section, it selects the **Input** and **Initial state** parameters and configures them to use the states and inputs in the exported operating point.

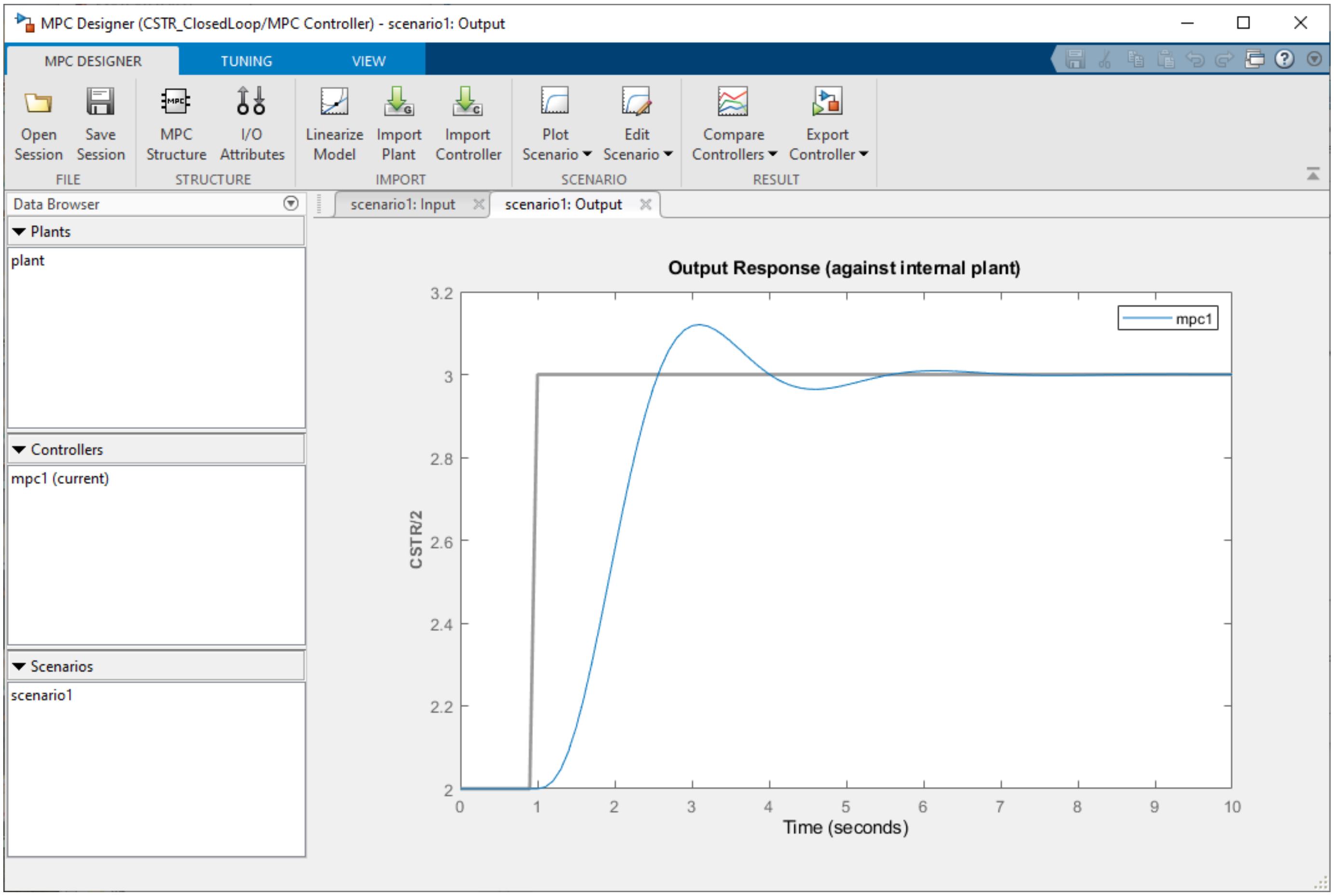
To reset the model initial conditions, for example if you delete the exported operating point, clear the **Input** and **Initial state** parameters.

Close the Edit dialog box.

In the Define MPC Structure By Linearization dialog box, linearize the model by clicking **Define and Linearize**.

In the **Data Browser**, the app adds the following items.

- Linearized plant model `plant`
- Default MPC controller `mpc1` created using the linearized plant as an internal prediction model
- Default simulation scenario `scenario1`

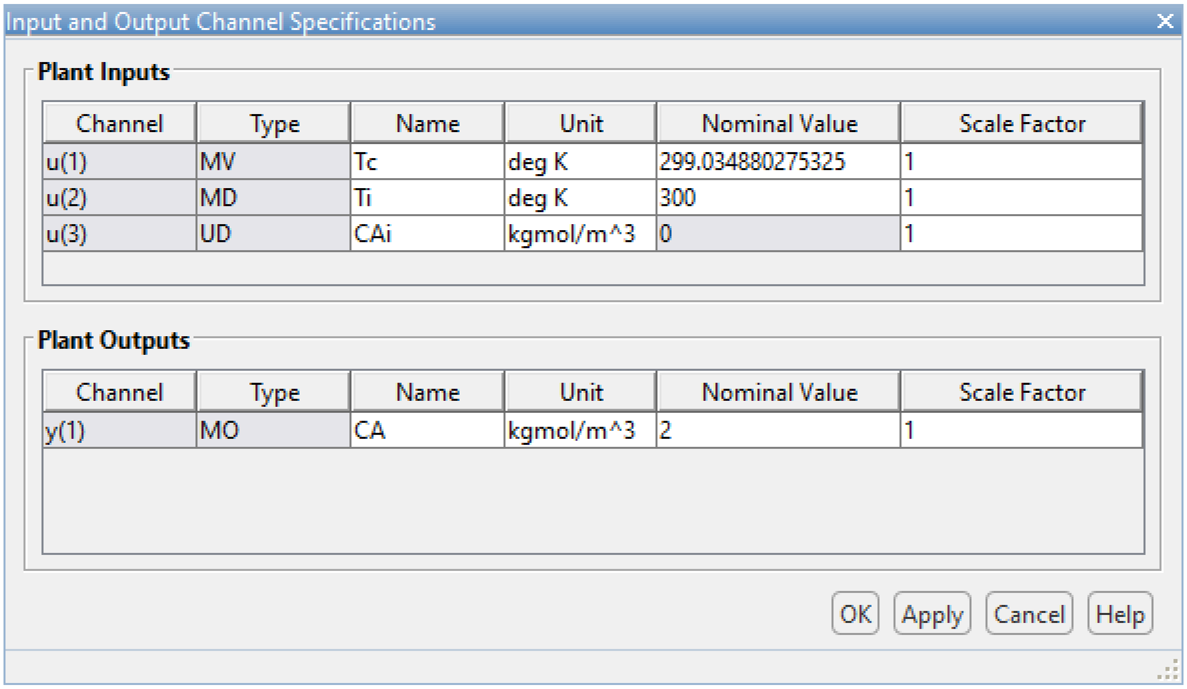


Define Input/Output Channel Attributes

On the **MPC Designer** tab, in the **Structure** section, click **I/O Attributes**.

In the Input and Output Channel Specifications dialog box, in the **Name** column, specify meaningful names for each input and output channel.

In the **Unit** column, specify appropriate units for each signal.



Input and Output Channel Specifications

Plant Inputs

Channel	Type	Name	Unit	Nominal Value	Scale Factor
u(1)	MV	Tc	deg K	299.034880275325	1
u(2)	MD	Ti	deg K	300	1
u(3)	UD	CAi	kgmol/m^3	0	1

Plant Outputs

Channel	Type	Name	Unit	Nominal Value	Scale Factor
y(1)	MO	CA	kgmol/m^3	2	1

The **Nominal Value** for each signal is the corresponding steady-state value at the computed operating point.

Click **OK**.

Define Disturbance Rejection Simulation Scenarios

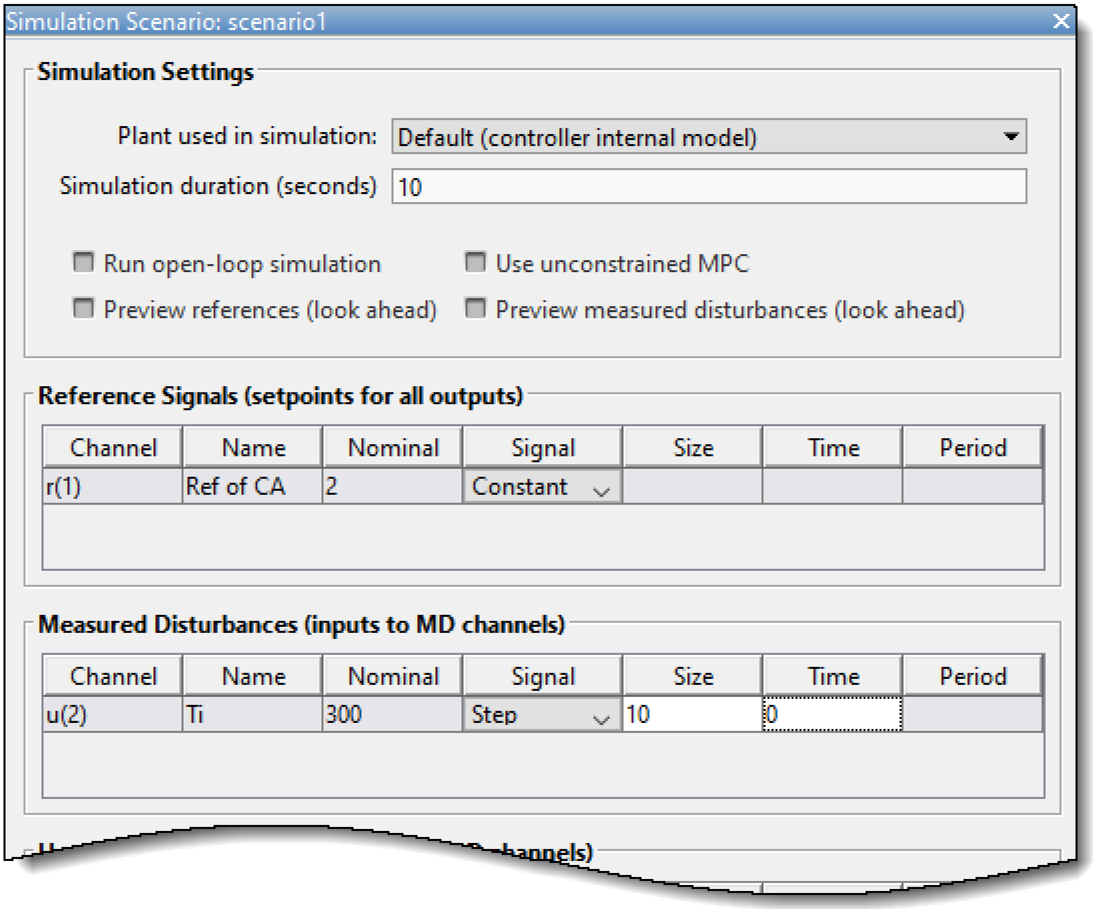
The primary objective of the controller is to hold the residual concentration *CA* at the nominal value of 2 kgmol/m^3. To do so, the controller must reject both measured and unmeasured disturbances.

On the **MPC Designer** tab, in the **Scenario** section, select **Edit Scenario** > **scenario1**.

In the Simulation Scenario dialog box, in the **Reference Signals** table, in the **Signal** drop-down list select **Constant** to hold the output setpoint at its nominal value.

In the **Measured Disturbances** table, in the **Signal** drop-down list, select **Step**.

Specify a step **Size** of `10` and a step **Time** of `0`.



Simulation Scenario: scenario1

Simulation Settings

Plant used in simulation: Default (controller internal model)

Simulation duration (seconds) 10

□ Run open-loop simulation □ Use unconstrained MPC

□ Preview references (look ahead) □ Preview measured disturbances (look ahead)

Reference Signals (setpoints for all outputs)

Channel	Name	Nominal	Signal	Size	Time	Period
r(1)	Ref of CA	2	Constant			

Measured Disturbances (inputs to MD channels)

Channel	Name	Nominal	Signal	Size	Time	Period
u(2)	Ti	300	Step	10	0	

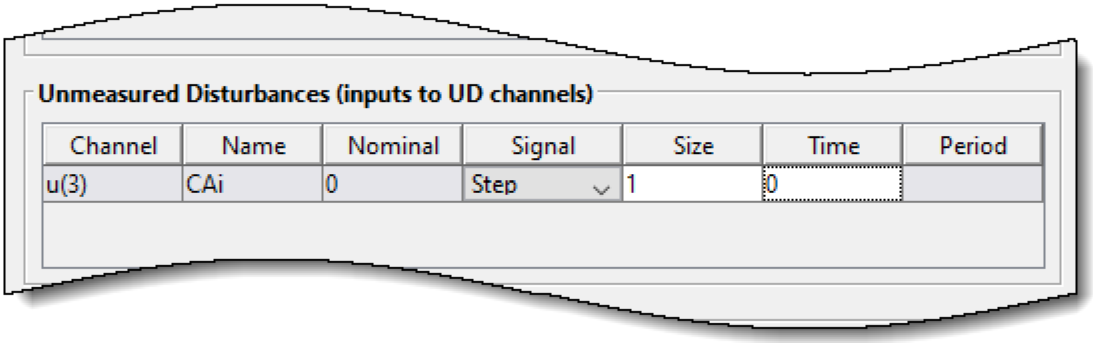
Click **OK**.

In the **Data Browser**, under **Scenarios**, click `scenario1`. Click `scenario1` a second time, and rename it `MD_reject`.

In the **Scenario** section, click **Plot Scenario** > **New Scenario**.

In the Simulation Scenario dialog box, in the **Unmeasured Disturbances** table, in the **Signal** drop-down list, select **Step**.

Specify a step **Size** of `1` and a step **Time** of `0`.



Unmeasured Disturbances (inputs to UD channels)

Channel	Name	Nominal	Signal	Size	Time	Period
u(3)	CAi	0	Step	1	0	

Click **OK**.

In the **Data Browser**, under **Scenarios**, rename `NewScenario` to `UD_reject`.

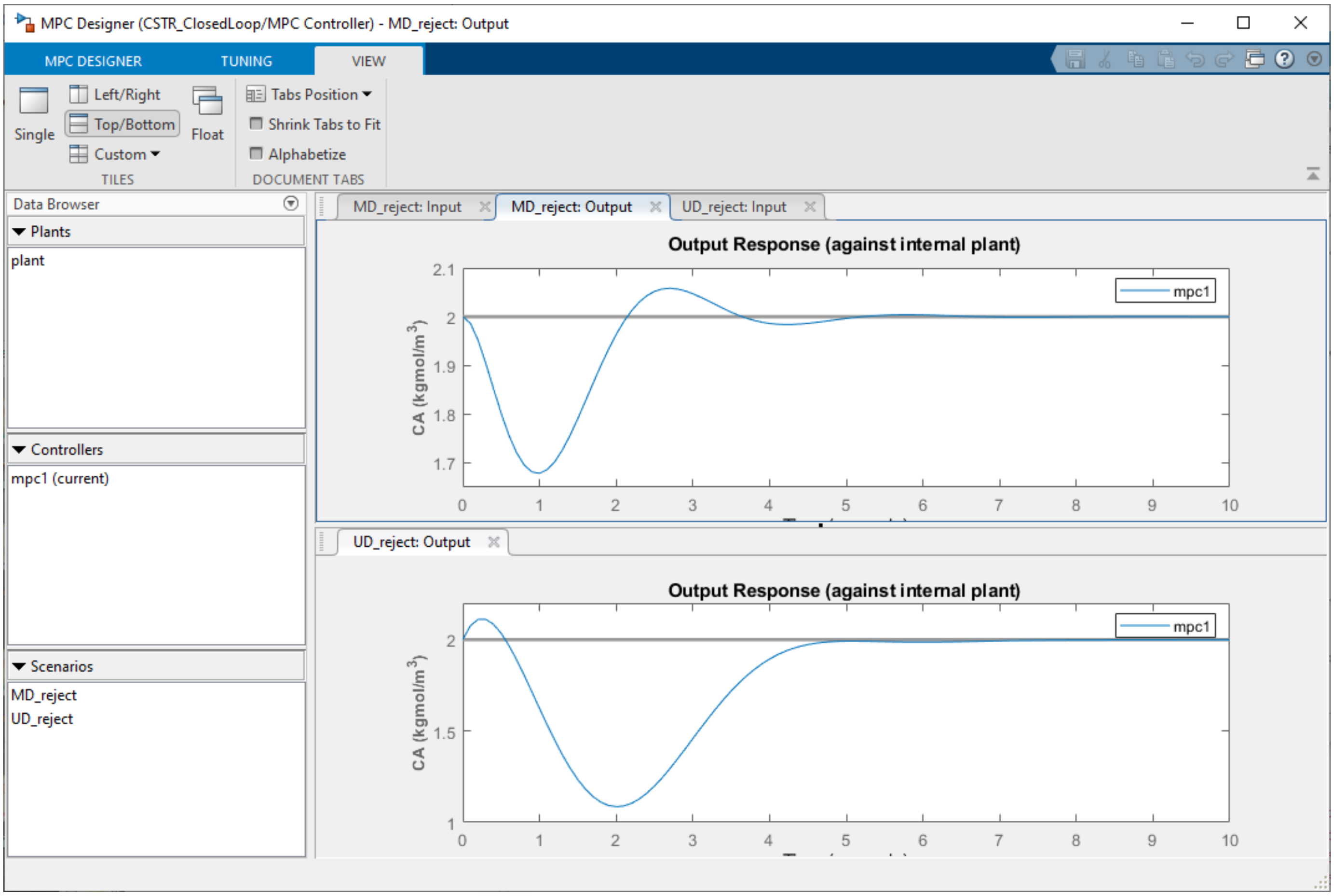
Arrange Output Response Plots

To make viewing the tuning results easier, arrange the plot area to display the Output Response plots for both scenarios at the same time.

On the **View** tab, in the **Tiles** section, click **Top/Bottom**. The **View** tab is not supported in MATLAB Online.

The plot display area changes to display the Input Response plots above the Output Response plots.

Drag and select the plots so that **MD_reject: Output** tab is in the upper plot area and the **UD_reject: Output** plot is in the lower plot area.



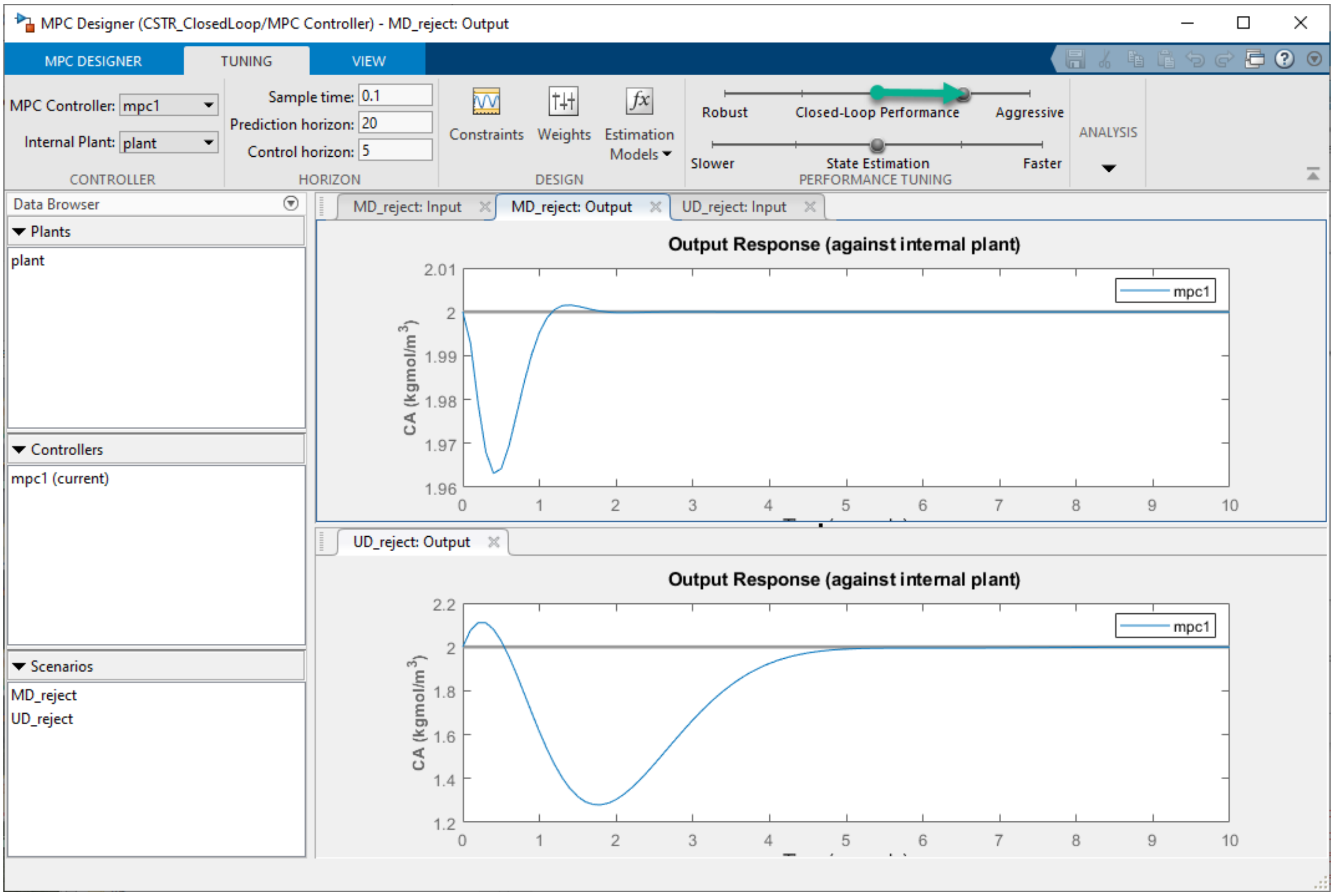
Tune Controller Performance

On the **Tuning** tab, in the **Horizon** section, specify a **Prediction horizon** of `20` and a **Control horizon** of `5`.

The **Output Response** plots update based on the new horizon values.

Use the default controller constraint and weight configurations.

In the **Performance Tuning** section, drag the **Closed-Loop Performance** slider to the right, which leads to tighter control of outputs and more aggressive control moves. Drag the slider until the **MD_reject: Output** response reaches steady state in less than two seconds.



MPC Designer (CSTR_ClosedLoop/MPC Controller) - MD_reject: Output

MPC Controller: mpc1

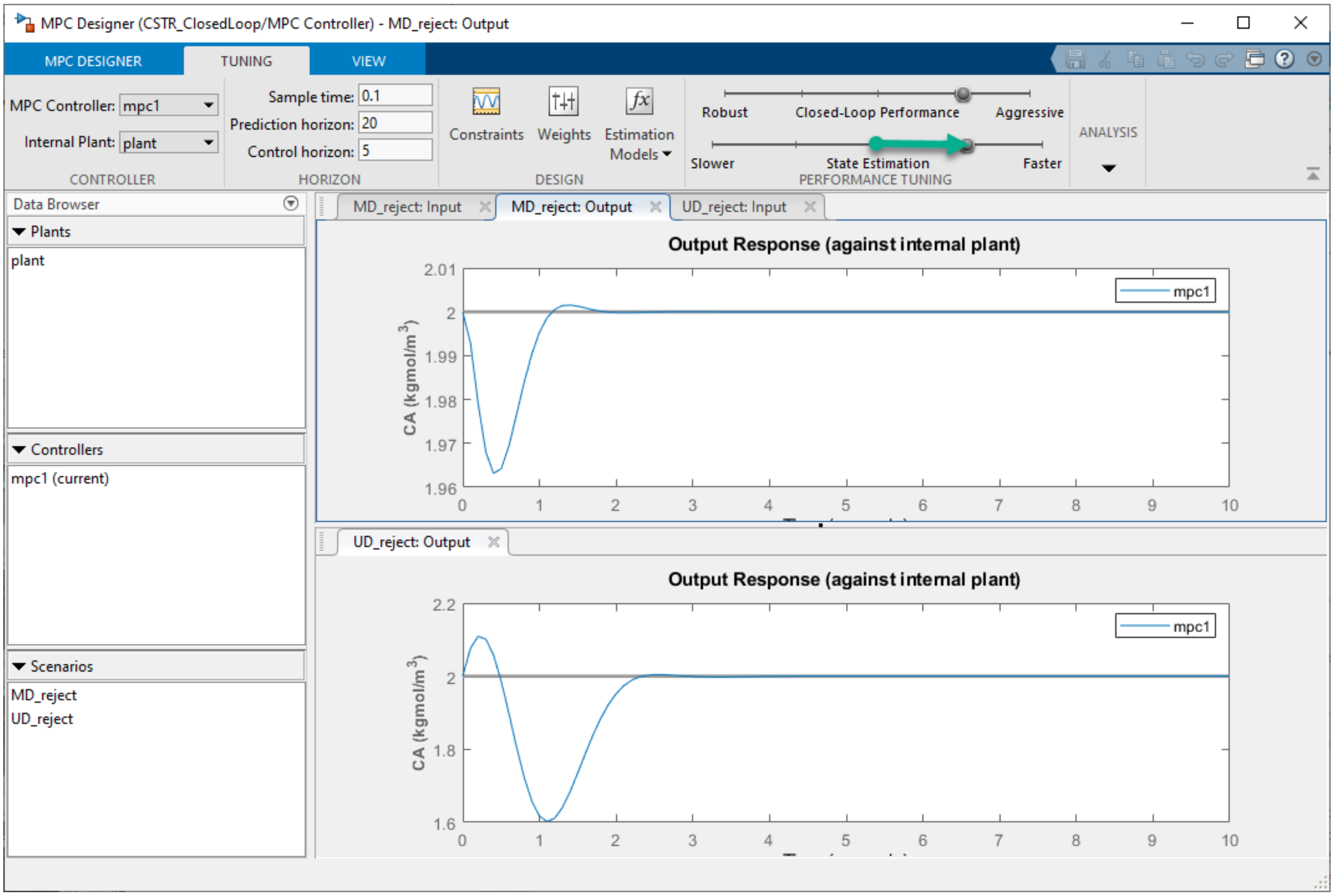
Internal Plant: plant

Sample time: 0.1

Prediction horizon: 20

Control horizon: 5

Drag the **State Estimation** slider to the right, which leads to more aggressive unmeasured disturbance rejection. Drag the slider until the **UD_reject: Output** response reaches steady state in less than 3 seconds.



MPC Controller: mpc1
Internal Plant: plant
Sample time: 0.1
Prediction horizon: 20
Control horizon: 5

Update Simulink Model with Tuned Controller

In the **Analysis** section, select **Export Controller** > **Update Block Only**. The app exports tuned controller `mpc1` to the MATLAB workspace. In the Simulink model, the MPC Controller block is updated to use the exported controller.

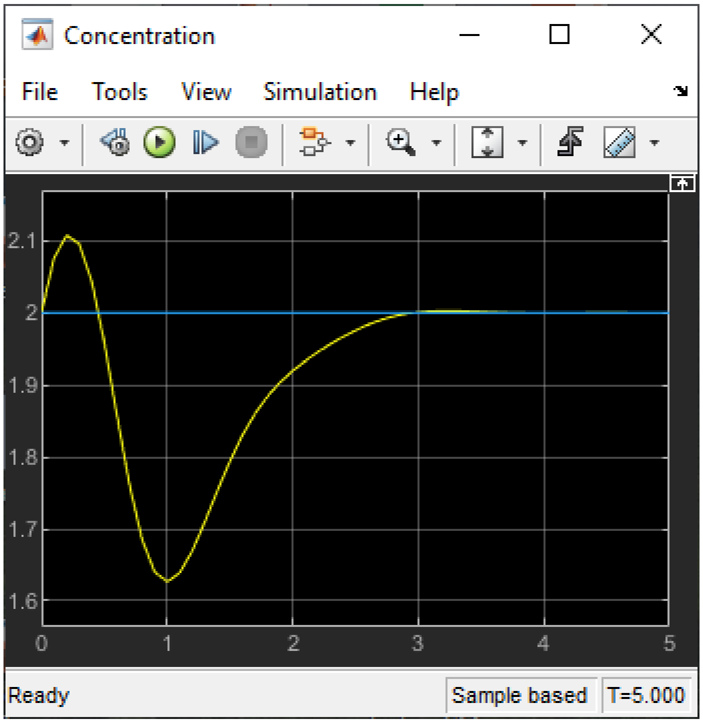
Simulate Unmeasured Disturbance Rejection

In the Simulink model window, on the **Simulation** tab, change **Stop Time** to `5` seconds.

The model initial conditions are set to the nominal operating point used for linearization.

To simulate a unit step in the feed concentration at time zero, open the Feed Concentration block and increase its **Constant value** parameter from `10` to `11`.

In the Simulink model window, open the Concentration scope and run the simulation.



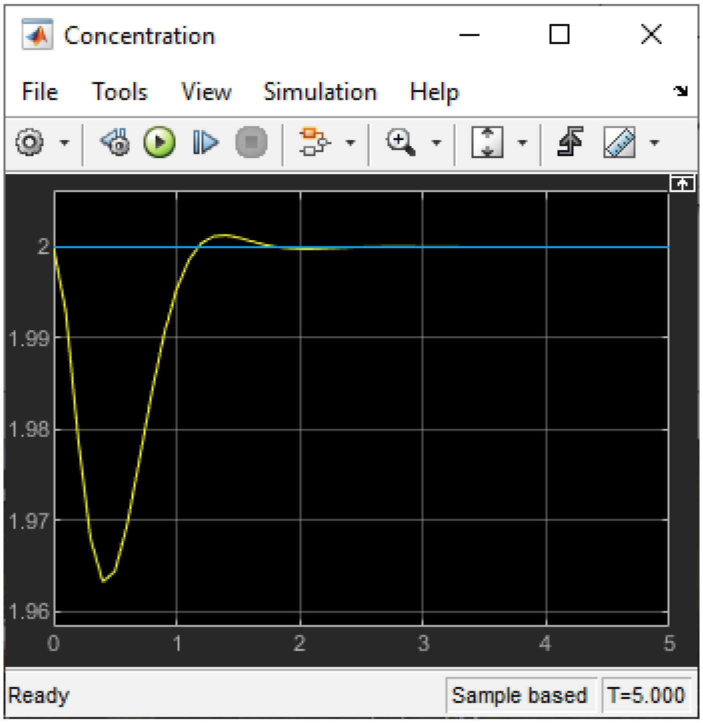
The output response is similar to the **UD_reject** response, however the settling time is around 1 second later. The different result is due to the mismatch between the linear plant used in the **MPC Designer** simulation and the nonlinear plant in the Simulink model.

Simulate Measured Disturbance Rejection

To simulate the measured disturbance rejection, first return the Feed Concentration block to its nominal value of `10`.

To simulate a step change in the feed temperature at time zero, open the Feed Temperature block and increase its **Constant value** parameter from `300` to `310`.

Run the simulation.



The output response is similar to the **MD_reject** response from the **MPC Designer** simulation.

References

[1] Seborg, Dale. E., Thomas. F. Edgar, and Duncan. A. Mellichamp, *Process Dynamics and Control*. 2nd ed. Hoboken, N.J: John Wiley & Sons, Inc, 2004. 34–36 and 94–95.

See Also

Apps
MPC Designer

Blocks
MPC Controller

More About

- “Tune Weights”
- “Linearize Simulink Models” on page 2-16
- “Design Controller Using MPC Designer” on page 3-2

Model Predictive Control of a Single-Input-Single-Output Plant

This example shows how to control a double integrator plant with input saturation in Simulink®.

Define the Plant Model

Define the plant model as a double integrator (the input is the manipulated variable and the output the measured output).

```
plant = tf(1,[1 0 0]);
```

Design the MPC Controller

Create the controller object with a sampling period of 0.1 seconds, a prediction horizon of 10 steps and a control horizon of and 3 moves.

```
mpcobj = mpc(plant, 0.1, 10, 3);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming default 0.00000
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming default 0.1
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1.00000.
```

Because you have not specified the weights of the quadratic cost function to be minimized by the controller, their value is assumed to be the default one (0 for the manipulated variables, 0.1 for the manipulated variable rates, 1 for the output variables). Also, at this point the MPC problem is still unconstrained as you have not specified any constraint yet.

Specify actuator saturation limits as constraints on the manipulated variable.

```
mpcobj.MV = struct('Min',-1,'Max',1);
```

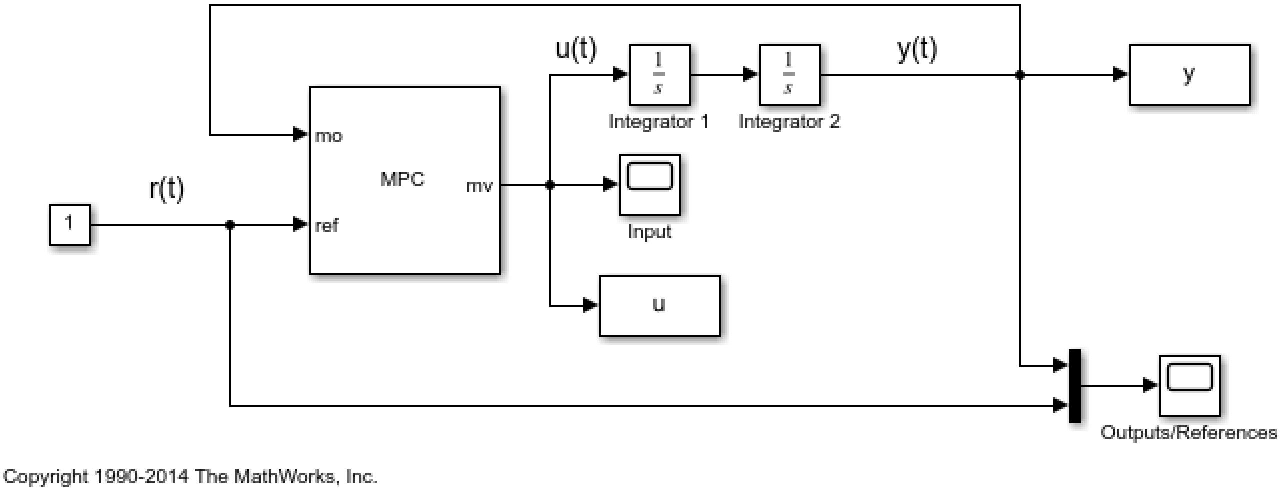
Simulate Using Simulink

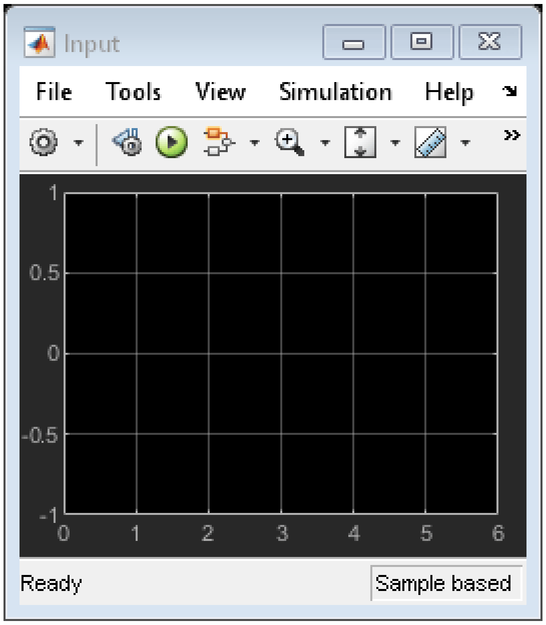
Simulink is a graphical block diagram environment for multidomain system simulation. You can connect blocks representing dynamical systems (in this case the plant and the MPC controller) and simulate the closed loop.

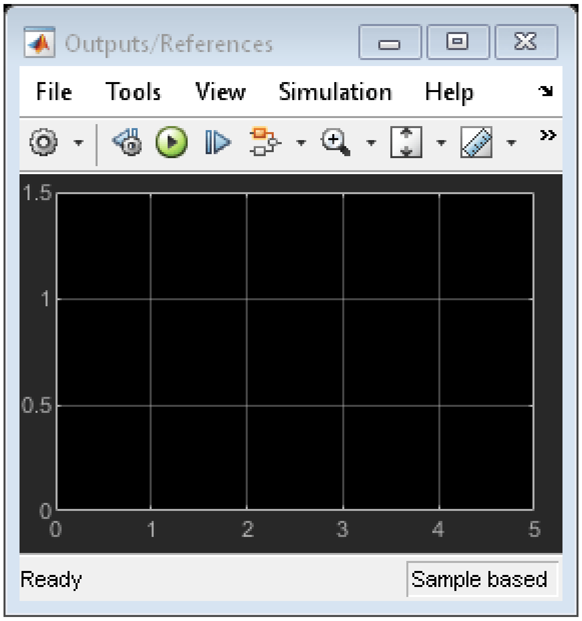
```
% Check that Simulink is installed, otherwise display a message and return
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink is required to run this example.')
    return
end
```

Open the pre-existing Simulink model for the closed-loop simulation. The plant model is implemented with two integrator blocks in series. The variable-step ode45 integration algorithm is used to calculate the continuous time loop behavior. The MPC Controller block is configured to use the workspace `mpcobj` object as controller. The manipulated variables and the output and reference signal. The output signal is also saved by the To-Workspace block.

```
mdl = 'mpc_doubleint';
open_system(mdl)
```


u(t)
y(t)
r(t)
1
mo
MPC
ref
mv
1/s
Integrator 1
1/s
Integrator 2
y
Input
u
Outputs/References
Copyright 1990-2014 The MathWorks, Inc.

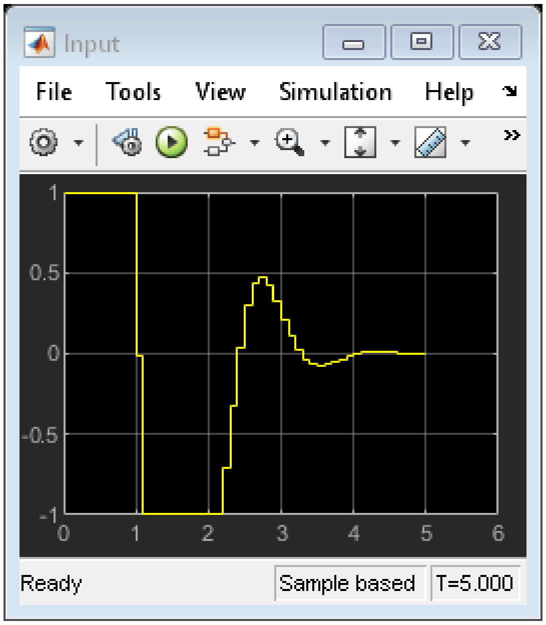

Input
File
Tools
View
Simulation
Help
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
Ready
Sample based

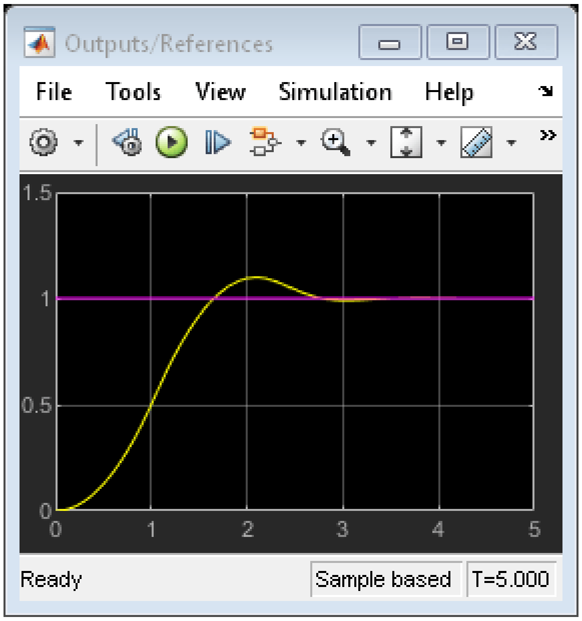


Simulate closed-loop control of the linear plant model in Simulink. Note that before the simulation starts the plant model in `mpcobj` is converted to a discrete state space model. By default, the controller uses as observer a Kalman filter designer assuming a white noise disturbance on each plant output.

```
sim(mdl)         % you can also simulate by pressing the "Run" button.
```

```
-->Converting the "Model.Plant" property of "mpc" object to state-space.
-->Converting model to discrete time.
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
```





The closed-loop response shows good setpoint tracking performance, as the plant output tracks its reference after about 2.5 seconds. As expected, the manipulated variable stays within the predefined constraints.

Close the open Simulink model without saving any change.

```
bdclose(mdl)
```

See Also

MPC Controller | **MPC Designer** | `mpc`

More About

- "Model Predictive Control of a Multi-Input Single-Output Plant" on page 3-51
- "Model Predictive Control of a Multi-Input Multi-Output Nonlinear Plant" on page 3-91

Model Predictive Control of a Multi-Input Single-Output Plant

This example shows how to design a model predictive controller for a plant with one measured output, one manipulated variable, one measured disturbance, and one unmeasured disturbance.

Define Plant Model

Define a plant model. For this example use continuous time transfer functions from each input to the output. Display the plant model at the command line.

```
plantTF = tf({1,1,1},{[1 .5 1],[1 1],[.7 .5 1]}) % display transfer functions
```

```
plantTF =

  From input 1 to output:
         1
  ---------------
  s^2 + 0.5 s + 1

  From input 2 to output:
    1
  -----
  s + 1

  From input 3 to output:
           1
  -------------------
  0.7 s^2 + 0.5 s + 1

Continuous-time transfer function.
```

For this example, explicitly convert the plant to a discrete-time state space form before passing in to the MPC controller creation function.

The controller creation function can accept either continuous-time or discrete-time plants. When the optimization problem to find the optimal value of the manipulated variable is set up, the MPC controller automatically converts a continuous-time plant (in any format) to a discrete-time state space model, using Zero Order Hold.

Converting the plant to discrete-time is useful when you need the discrete-time system matrices for analysis or simulation (as in this example) or want the controller to use a discrete-time plant converted with a method other than ZOH.

```
plantCSS = ss(plantTF);         % convert plant from transfer function to continuous-time state s
Ts = 0.2;                       % specify a sample time of 0.2 seconds
plantDSS = c2d(plantCSS,Ts);    % convert plant to discrete-time state space, using Zero Order Ho
```

By default, all the plant input signals are assumed to be manipulated variables. Use `setmpcsignal` to specify which signals are measured and unmeasured disturbances. In this example, the first input signal is a manipulated variable, the second is a measured disturbance, the third one is an unmeasured disturbance. This information is stored in the plant model `plantDSS` and later used by the mpc controller.

```
plantDSS = setmpcsignals(plantDSS,'MV',1,'MD',2,'UD',3); % specify signal types
```

Design MPC Controller

Create the controller object, specifying the sampling time, as well as the prediction and control horizons (10 and 3 steps respectively).

```
mpcobj = mpc(plantDSS,Ts,10,3);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming default 0.00000
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming default 0.1
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1.00000.
```

Because you have not specified the weights of the quadratic cost function to be minimized by the controller, their value is assumed to be the default one (0 for the manipulated variables, 0.1 for the manipulated variable rates, 1 for the output variables). Also, at this point the MPC problem is still unconstrained as you have not specified any constraint yet.

Define hard constraints on the manipulated variable.

```
mpcobj.MV = struct('Min',0,'Max',1,'RateMin',-10,'RateMax',10);
```

The input and output disturbance models specify the dynamic characteristics of the unmeasured disturbances on the input and output, respectively, so they can be better rejected. By default, these disturbance models are assumed to be integrators unless you specify them otherwise. The mpc object also has a noise model that specifies the dynamics of the noise on the measured output variable. By default this is assumed to be a unity static gain, which is equivalent to assume that the noise is white. In this example, there is no measured output disturbance, so there is no need to specify an output disturbance model, and the noise on the measured output signal is assumed to be white.

Specify the disturbance model for the unmeasured input as an integrator driven by white noise with variance = `1000`.

```
mpcobj.Model.Disturbance = tf(sqrt(1000),[1 0]);
```

Display the MPC controller object `mpcobj` to review its properties.

```
mpcobj
```

```
MPC object (created on 24-Feb-2021 00:13:11):
---------------------------------------------
Sampling time:      0.2 (seconds)
Prediction Horizon: 10
Control Horizon:    3

Plant Model:
                                      --------------
      1  manipulated variable(s)   -->|  5 states  |
                                      |            |-->  1 measured output(s)
      1  measured disturbance(s)   -->|  3 inputs  |
                                      |            |-->  0 unmeasured output(s)
      1  unmeasured disturbance(s) -->|  1 outputs |
                                      --------------
Indices:
  (input vector)    Manipulated variables: [1 ]
                     Measured disturbances: [2 ]
                  Unmeasured disturbances: [3 ]
  (output vector)        Measured outputs: [1 ]
```

```
Disturbance and Noise Models:
        Output disturbance model: default (type "getoutdist(mpcobj)" for details)
         Input disturbance model: user specified (type "getindist(mpcobj)" for more details)
         Measurement noise model: default (unity gain after scaling)

Weights:
        ManipulatedVariables: 0
    ManipulatedVariablesRate: 0.1000
             OutputVariables: 1
                         ECR: 100000

State Estimation:  Default Kalman Filter (type "getEstimator(mpcobj)" for details)

Constraints:
 0 <= MV1 <= 1, -10 <= MV1/rate <= 10, MO1 is unconstrained
```

Display the input disturbance model. As expected is the specified integrator converted to discrete time.

```
getindist(mpcobj)
```

```
ans =

  A = 
       x1
   x1   1

  B = 
       Noise#1
   x1      0.8

  C = 
           x1
   UD1  7.906

  D = 
        Noise#1
   UD1        0

Sample time: 0.2 seconds
Discrete-time state-space model.
```

Display the output disturbance model. As expected it is empty.

```
getoutdist(mpcobj)
```

```
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured

ans =

  Empty state-space model.
```

Examine Steady-State Offset

To examine whether the MPC controller will be able to reject constant output disturbances and track constant setpoint with zero offsets in steady-state, compute the closed loop DC gain from output disturbances to controlled outputs using the `cloffset` command. This is also known as the steady state output sensitivity of the closed loop.

```
DC = cloffset(mpcobj);
fprintf('DC gain from output disturbance to output = %5.8f (=%g) \n',DC,DC);
```

```
DC gain from output disturbance to output = 0.00000000 (=3.21965e-15)
```

A zero gain (which is typically the result of the controller having integrators as input or output disturbance models) means that the measured plant output will track the desired output reference setpoint.

Simulate Closed-Loop Response Using the `sim` Command

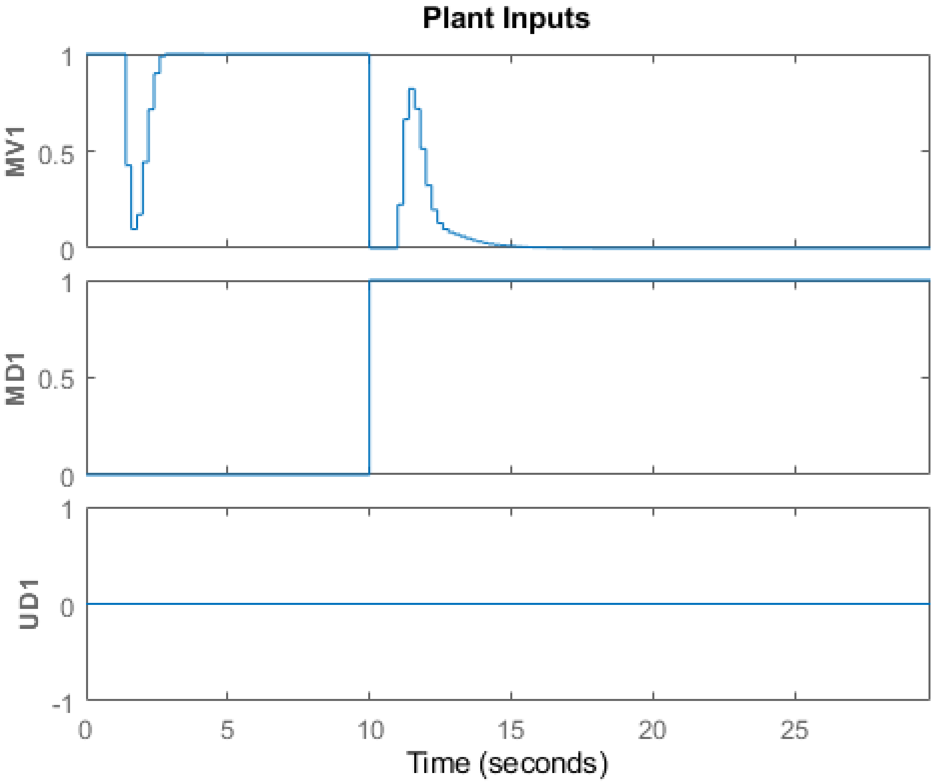
The `sim` command provides a quick way to simulate an MPC controller in a closed loop with a linear time-invariant plant when constraints and weights stay constants and the disturbance and reference signals can be easily and completely specified ahead of time.

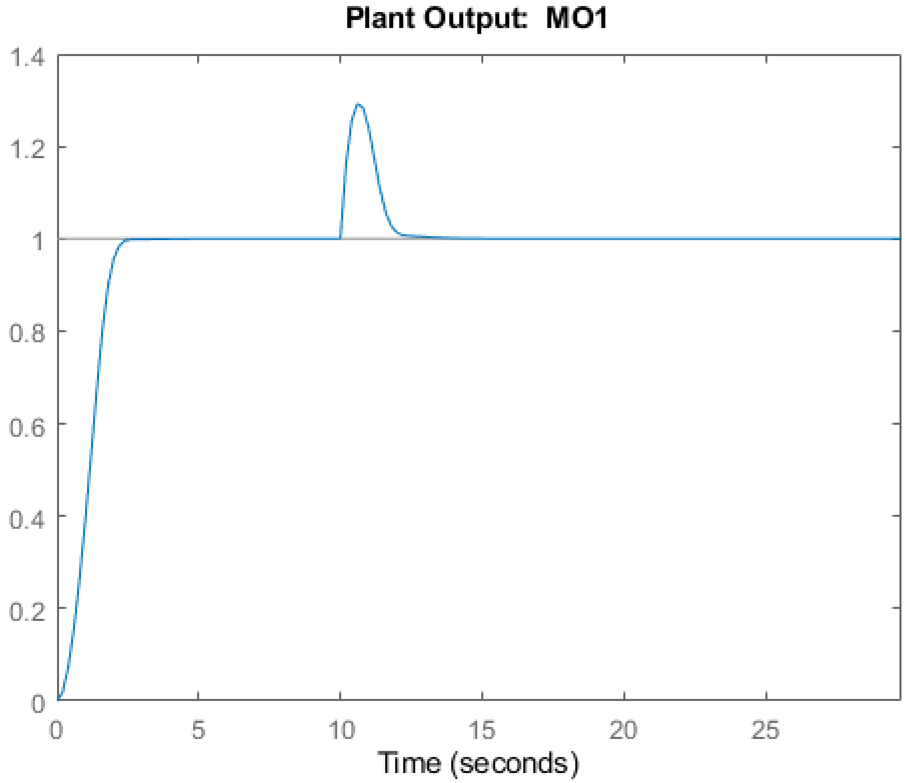
First, specify simulation time and the reference and disturbance signals

```
Tstop = 30;                               % simulation time
Nf = round(Tstop/Ts);                     % number of simulation steps
r = ones(Nf,1);                           % output reference signal
v = [zeros(Nf/3,1);ones(2*Nf/3,1)];       % measured input disturbance signal
```

Run the closed-loop simulation and plot results. The plant specified in `mpcobj.Model.Plant` is used both as a plant model in the closed loop and as the internal plant model used by the controller to predict the response over the prediction horizon. Use `sim` to simulate the closed loop system for Nf steps with reference r and measured input disturbance v.

```
sim(mpcobj,Nf,r,v)      % simulate plant and controller in closed loop
```


Plant Inputs
MV1
1
0.5
0
MD1
1
0.5
0
UD1
1
0
-1
0
5
10
15
20
25
Time (seconds)



The manipulated variable hits the upper bound initially, and brings the plant output to the reference value within a few seconds. It then settles in at its maximum allowed value, 1. After 10 seconds, the measured disturbance signal rises from 0 to 1, which causes the plant output to exceed its reference value by about 30%. The manipulated variable hits the lower bound in an effort to reject the disturbance. The controller is able to bring the plant output back to the reference value after a few seconds, and the manipulated variable settles in at its minimum value. The unmeasured disturbance signal is always zero, because no unmeasured disturbance has been specified yet.

You can use a simulation options objects to specify additional simulation options as well as noise and disturbance signals that feed into the plant but are unknown to the controller. For this example, use a simulation option object to specify the unmeasured input disturbance signal as well as additive noise on both manipulated variables and measured outputs. You will also later use this options object to specify a plant to be used in simulation, which differs from the one used internally by the controller. Create a simulation option object with default options and no signal.

```
SimOptions = mpcsimopt;                         % create object
```

Create a disturbance signal and specify it in the simulation options object

```
d = [zeros(2*Nf/3,1);-0.5*ones(Nf/3,1)];        % define a step disturbance signal
SimOptions.UnmeasuredDisturbance = d;           % specify unmeasured input disturbance signal
```

Specify noise signals in the simulation options object. At simulation time, the simulation function directly adds the specified output noise to the measured output before feeding it to the controller. It also directly adds the specified input noise to the manipulated variable (not to any disturbance signals) before feeding it to the plant.

```
SimOptions.OutputNoise=.001*(rand(Nf,1)-.5);    % specify output measurement noise
SimOptions.InputNoise=.05*(rand(Nf,1)-.5);      % specify noise on manipulated variables
```

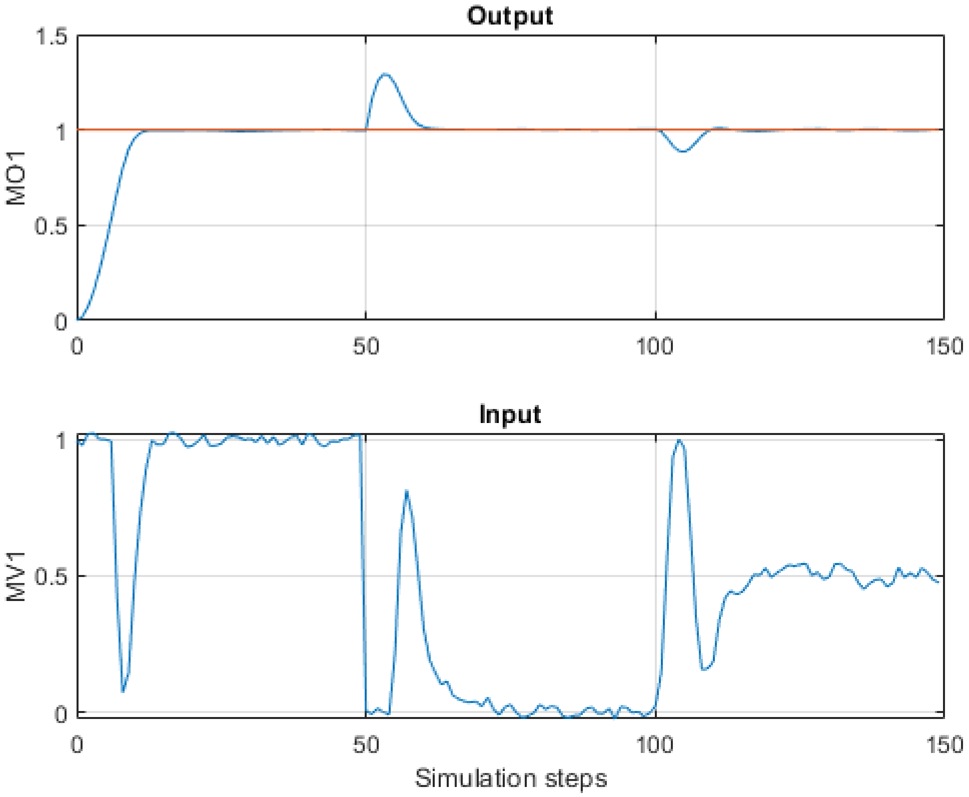
Note that you can also use the `OutputNoise` field of the simulation option object to specify a more general additive output disturbance signal (such as a step, for example) on the measured plant output.

Use `sim` with the additional `SimOptions` argument to simulate the closed loop system and save the results to the workspace variables `y`,|t|,|u|, and `xp`. This allows you to selectively plot signals in a new figure window and in any given color and order.

```
[y,t,u,xp] = sim(mpcobj,Nf,r,v,SimOptions);     % simulate closed loop
```

Plot results.

```
figure                                  % create new figure
subplot(2,1,1)                          % create upper subplot
plot(0:Nf-1,y,0:Nf-1,r)                 % plot plant output and reference
title('Output')                         % add title so upper subplot
ylabel('MO1')                           % add a label to the upper y axis
grid                                    % add a grid to upper subplot
subplot(2,1,2)                          % create lower subplot
plot(0:Nf-1,u)                          % plot manipulated variable
title('Input');                         % add title so lower subplot
xlabel('Simulation steps')              % add a label to the lower x axis
ylabel('MV1')                           % add a label to the lower y axis
grid                                    % add a grid to lower subplot
```



Despite the added noise (which is especially visible on the manipulated variable plot) and despite the measured and unmeasured disturbances kicking in after 50 and 100 steps respectively, the controller is able to achieve and maintain good tracking. The manipulated variable settles in at about 1 after the initial part of the simulation (steps from 20 to 50), at about 0 to reject the measured disturbance (steps from 70 to 100), and at about 0.5 to reject both disturbances (steps from 120 to 150).

Simulate Closed-Loop Response with Model Mismatch

Test the robustness of the MPC controller against a model mismatch. Specify the true plant to be used in simulation as `truePlantCSS`.

```
truePlantTF = tf({1,1,1},{[1 .8 1],[1 2],[.6 .6 1]})    % specify and display transfer functions
truePlantCSS = ss(truePlantTF);                         % convert to continuous state space
truePlantCSS = setmpcsignals(truePlantCSS,'MV',1,'MD',2,'UD',3); % specify signal types
```

```
truePlantTF =

  From input 1 to output:
         1
  ---------------
  s^2 + 0.8 s + 1

  From input 2 to output:
    1
  -----
  s + 2

  From input 3 to output:
           1
  -------------------
  0.6 s^2 + 0.6 s + 1

Continuous-time transfer function.
```

Update the simulation option object by specifying `SimOptions.Model` as a structure with two fields, `Plant` (containing the true plant model) and `Nominal` (containing the operating point values for the true plant). For this example, the nominal values for the state derivatives and the inputs are not specified, so they are assumed to be zero, resulting in `y = SimOptions.Model.Nominal.Y + C*(x-SimOptions.Model.Nominal.X)` where `x` and `y` are the state and measured output of the plant, respectively.

```
SimOptions.Model = struct('Plant',truePlantCSS);    % create the structure and assign the 'Plant
SimOptions.Model.Nominal.Y = 0.1;                   % create and assign the 'Nominal.Y' field
SimOptions.Model.Nominal.X = -.1*[1 1 1 1 1];       % create and assign the 'Nominal.X' field
SimOptions.PlantInitialState = [0.1 0 -0.1 0 .05];  % specify the initial state of the true plan
```

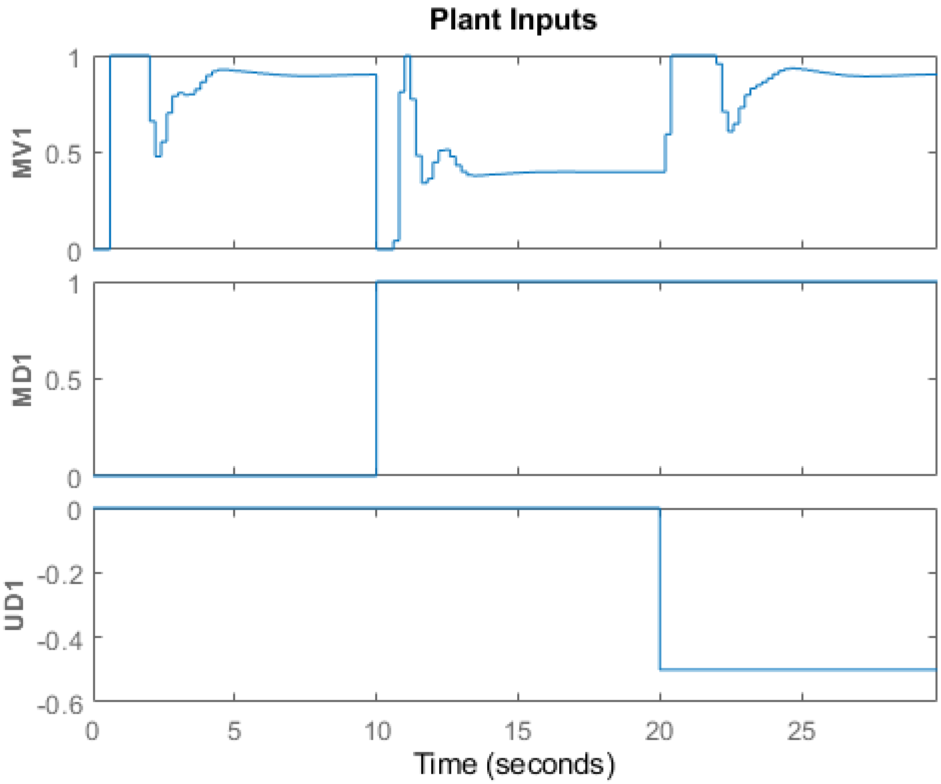
remove any signal previously added to the measured output and to the manipulated variable

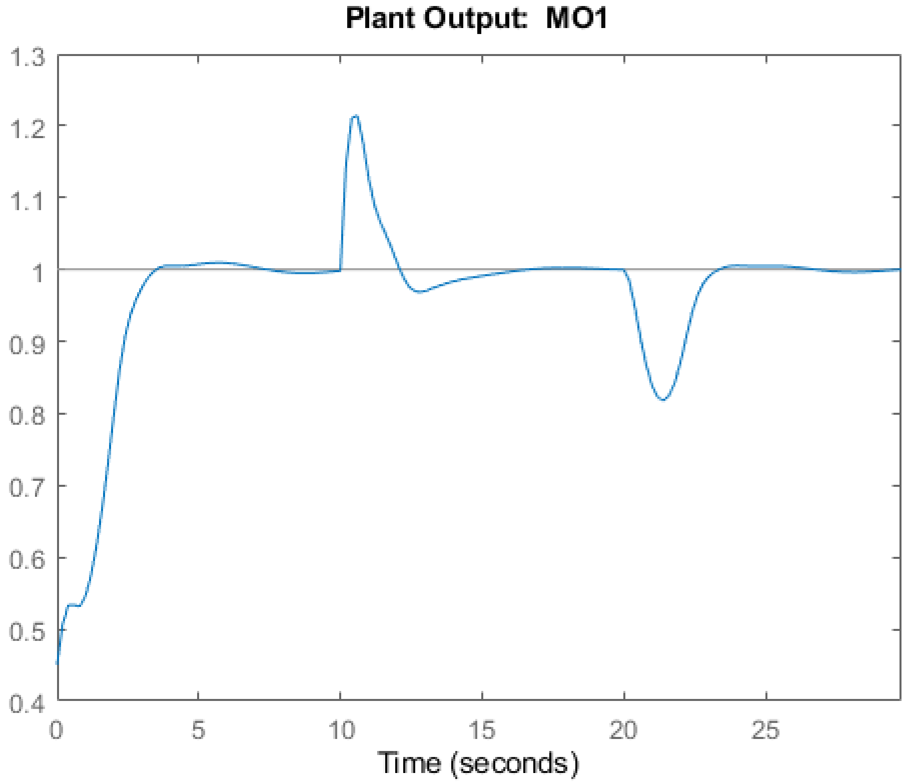
```
SimOptions.OutputNoise = [];                        % remove output measurement noise
SimOptions.InputNoise = [];                         % remove noise on manipulated variable
```

Run the closed-loop simulation and plot the results. Since `SimOptions.Model` is not empty, `SimOptions.Model.Plant` is converted to discrete time (using zero order hold) and used as the plant model in the closed loop simulation, while the plant in `mpcobj.Model.Plant` is only used by the controller to predict the response over the prediction horizon.

```
sim(mpcobj,Nf,r,v,SimOptions)       % simulate the closed loop
```

```
-->Converting model to discrete time.
```





As a result of the model mismatch, some degradation in the response is visible, notably the controller needs a little more time to achieve tracking and the manipulated variable now settles at about 0.5 to reject the measured disturbance (see values from 5 to 10 seconds) and settles at about 0.9 to reject both input disturbances (from 25 to 30 seconds). Despite this, the controller is still able to track the output reference.

Soften Constraints

Every constraint is associated to a dimensionless parameter called ECR value. A constraint with larger ECR value is allowed to be violated more than a constraint with smaller ECR value. By default all constraints on the manipulated variables have an ECR value of zero, making them hard. You can specify a nonzero ECR value for a constraint to make it soft.

Note that is possible to refer to the `ManipulatedVariables` field also as `MV`. Relax the constraints on manipulated variables from hard to soft.

```
mpcobj.MV.MinECR = 1;   % ECR for the lower bound on the manipulated variable
mpcobj.MV.MaxECR = 1;   % ECR for the upped bound on the manipulated variable
```

Define an output constraint. By default all constraints on output variables (measured outputs) have an ECR value of one, making them soft. You can reduce the ECR value for an output constraint to make it harder, however it is always advised to keep output constraints soft. This is mainly because the plant outputs depend on plant states as well as on the measured disturbances, therefore, if a large disturbance occurs, the plant outputs can violate the constraints independently on any control action taken by the mpc controller, especially when the manipulated variables are themselves hard bounded. Such an unavoidable violation of an hard constraint results in an infeasible MPC problem, for which no MV can be calculated.

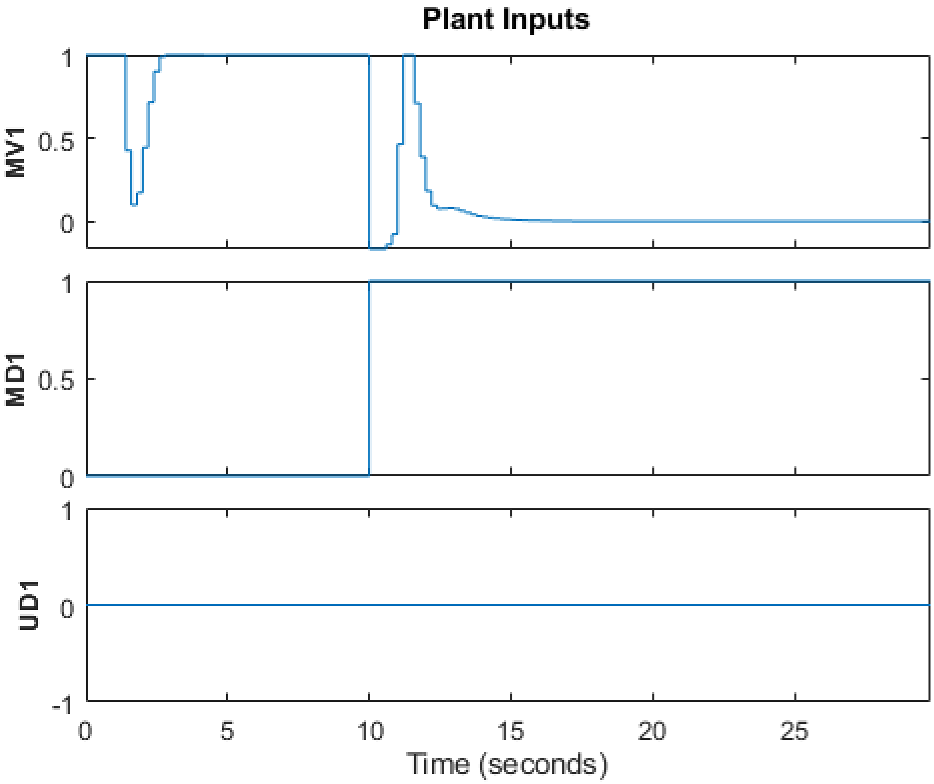
```
mpcobj.OV.Max = 1.1;    % define the (soft) output constraint
```

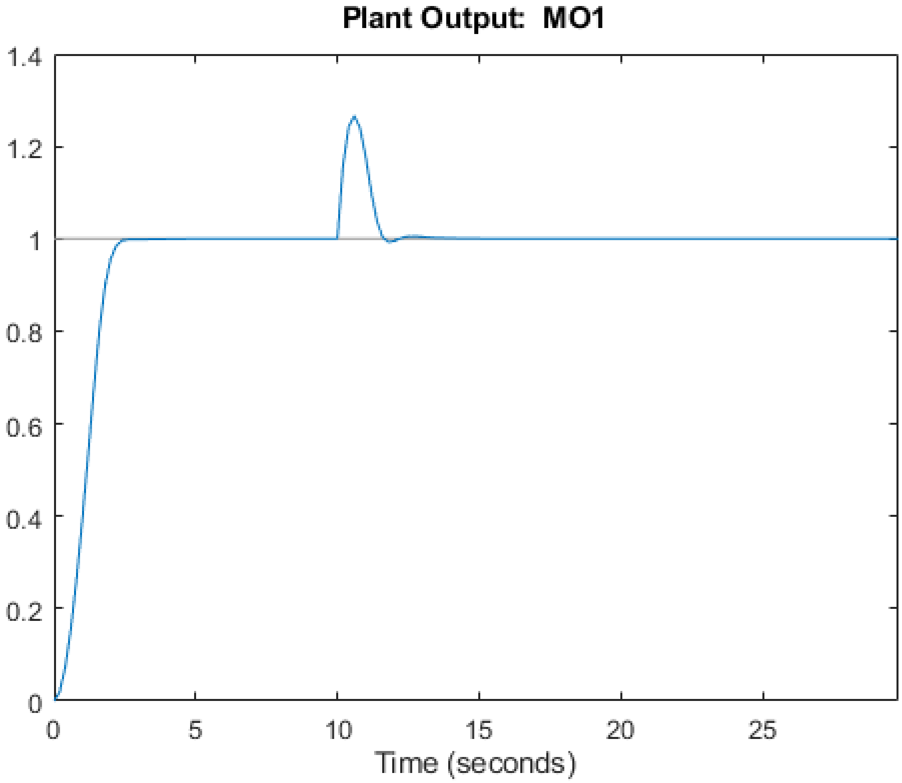
```
% Note that is possible to refer to the |OutputVariables| field also as |OV|.
```

Run a new closed-loop simulation, without including the simulation option object, and therefore without any model mismatch, unmeasured disturbance, or noise added to the manipulated variable or measured output.

```
sim(mpcobj,Nf,r,v)          % simulate the closed loop
```

```
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
```



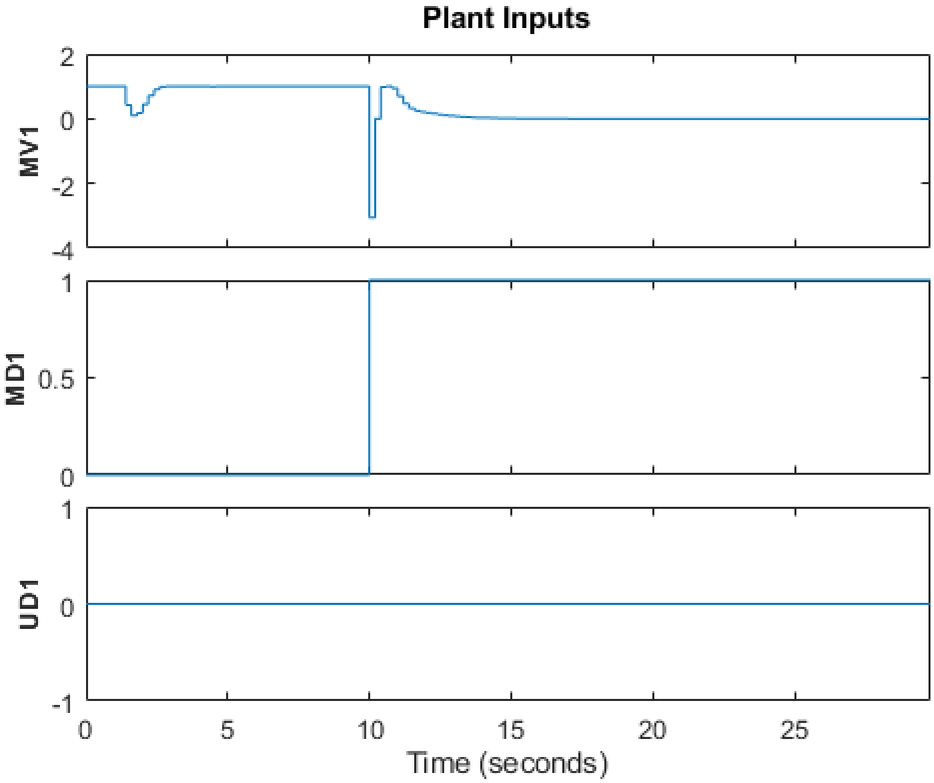


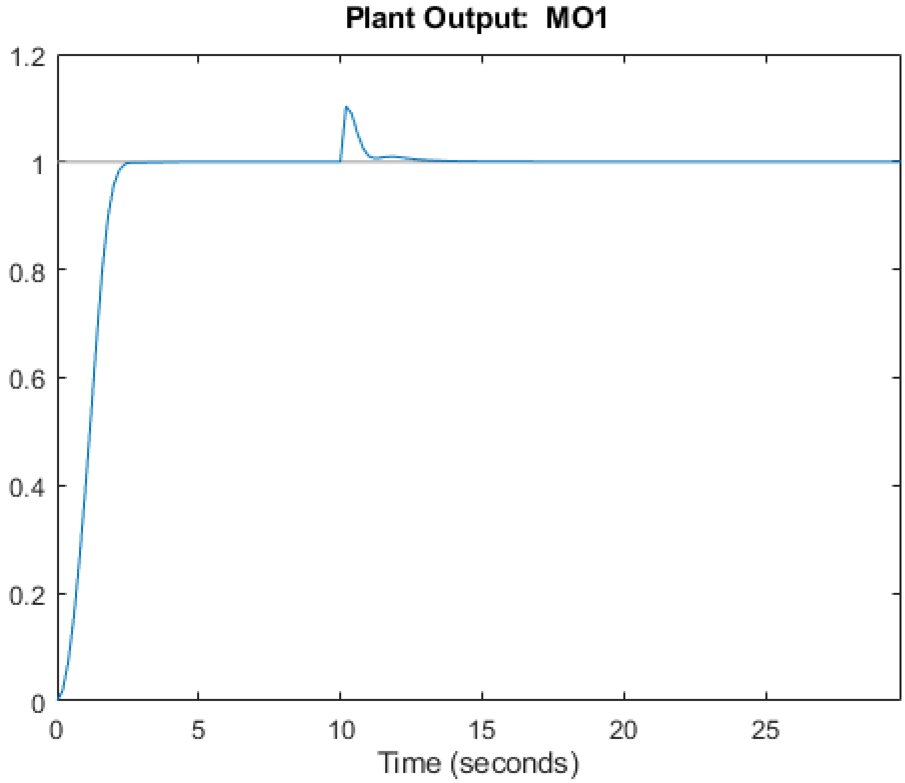
In an effort to reject the measured disturbance, achieve tracking, and prevent the output to rises above its 1.1 soft constraint, the controller slightly violates the soft constraint on the manipulated variable, which reaches small negative values from seconds 10 to 11 (you can zoom in the picture to see this violation more clearly). The constraint on the measured output is violated more than the one on the manipulated variable.

Penalize more the output constraint violations and rerun the simulation.

```
mpcobj.OV.MaxECR = 0.001;   % the closer to zero, the harder the constraint
sim(mpcobj,Nf,r,v)          % run a new closed-loop simulation.
```

```
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
```


Plant Inputs
MV1
2
0
-2
-4
MD1
1
0.5
0
UD1
1
0
-1
0
5
10
15
20
25
Time (seconds)



Now the controller violates the output constraint only very slighly. As expected, this output performance improvement comes at the cost of violating the manipulated variable constraint a lot more (the manipulated variable reaches -3 for a couple of steps).

Change Kalman Gains Used in the Built-In State Estimator

At each time step, the MPC controller obtains the manipulated variable by multiplying the discrete-time plant state (if available) by a gain matrix (computed by solving a constrained quadratic optimization problem). Since the plant state is normally not available, by default, the controller uses a linear Kalman filter as an observer to estimate the state of the discrete-time augmented plant (that is the plant comprehensive of the disturbance and noise models). Therefore, the states of the controller are the states of this Kalman filter, which are in turn the estimates of the states of the augmented discrete-time plant.

Run a closed loop simulation with model mismatch and unmeasured disturbance, using the default estimator, and return the controller states structure xc in the workspace, so you can later compare the state sequences.

```
[y,t,u,xp,xc] = sim(mpcobj,Nf,r,v,SimOptions);
```

```
-->Converting model to discrete time.
```

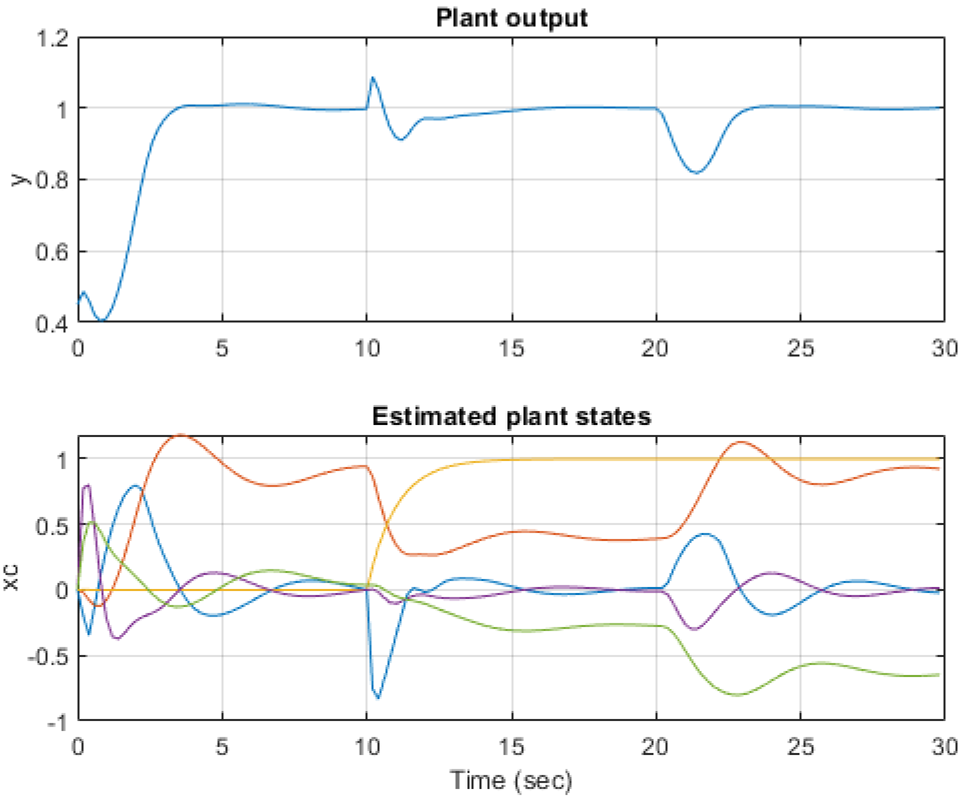
display the component of the controller states structure

```
xc
```

```
xc =
  struct with fields:

           Plant: [150x5 double]
     Disturbance: [150x1 double]
           Noise: [150x0 double]
        LastMove: [150x1 double]
```

Plot the plant output response as well as the plant states estimated by the default observer

```
figure;                                    % create figure
subplot(2,1,1)                             % create upper subplot axis
plot(t,y)                                  % plot y versus time
title('Plant output');                     % add title to upper plot
ylabel('y')                                % add a label to the upper y axis
grid                                       % add grid to upper plot
subplot(2,1,2)                             % create lower subplot axis
plot(t,xc.Plant)                           % plot xc.Plant versus time
title('Estimated plant states');           % add title to lower plot
xlabel('Time (sec)')                       % add a label to the lower x axis
ylabel('xc')                               % add a label to the lower y axis
grid                                       % add grid to lower plot
```



As expected, there are sudden changes at 10 and 20 seconds due to the measured and unmeasured disturbance signals kicking in, respectively.

You can change the gains of the Kalman filter used as observer. To do so, first, retrieve the default Kalman gains and state-space matrices.

```
[L,M,A1,Cm1] = getEstimator(mpcobj);    % retrieve observer matrices
```

Calculate and display the poles of the default observer. Note that, as expected, they are all inside the unit circle, though a few of them seem relatively close to the border. Note that there are six states, five belonging to the plant model, the sixth belonging to the input disturbance model.

```
e = eig(A1-A1*M*Cm1)                    % eigenvalues of observer state matrix
```

```
e =

   0.5708 + 0.4144i
   0.5708 - 0.4144i
   0.4967 + 0.0000i
   0.9334 + 0.1831i
   0.9334 - 0.1831i
   0.8189 + 0.0000i
```

Design a new state estimator by pole-placement.

```
poles = [.8 .75 .7 .85 .6 .81];         % specify desired positions for the new poles
L = place(A1',Cm1',poles)';             % calculate Kalman gain for time update
M = A1\L;                               % calculate Kalman gain for measurement update
```

Set the new matrix gains in the mpc controller object

```
setEstimator(mpcobj,L,M);               % set the new estimation gains
```

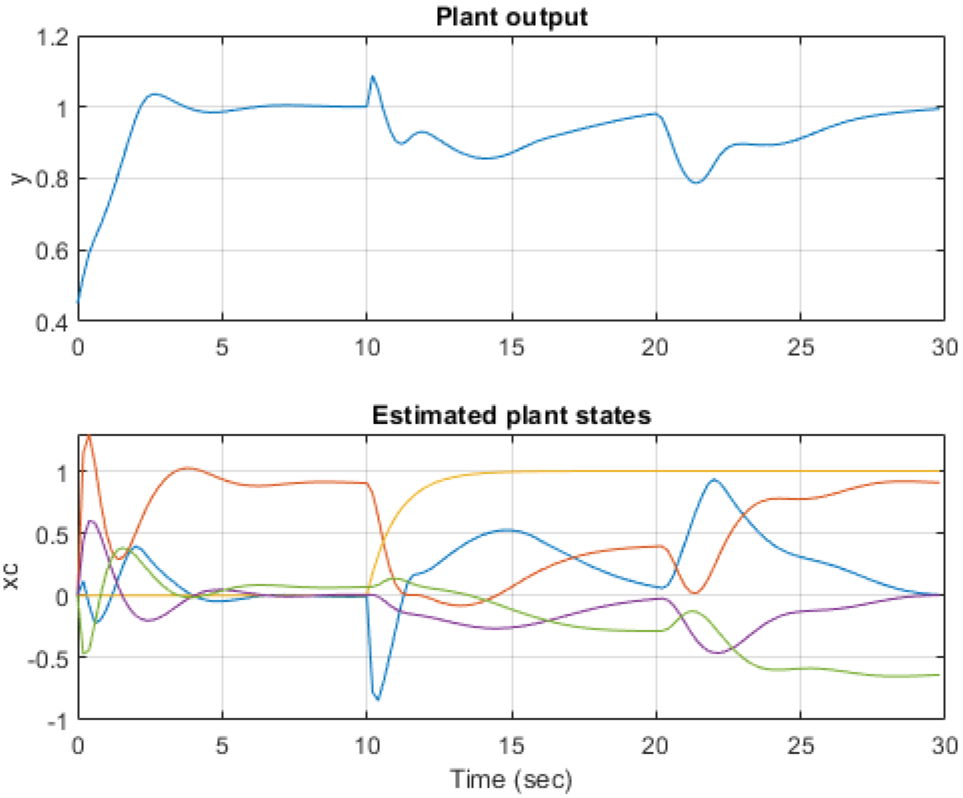
re-run the closed loop simulation

```
[y,t,u,xp,xc] = sim(mpcobj,Nf,r,v,SimOptions);
```

```
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
-->Converting model to discrete time.
```

Plot the plant output response as well as the plant states estimated by the new observer

```
figure;                                     % create figure
subplot(2,1,1)                              % create upper subplot axis
plot(t,y)                                   % plot y versus time
title('Plant output');                      % add title to upper plot
ylabel('y')                                 % add a label to the upper y axis
grid                                        % add grid to upper plot
subplot(2,1,2)                              % create lower subplot axis
plot(t,xc.Plant)                            % plot xc.Plant versus time
title('Estimated plant states');            % add title to lower plot
xlabel('Time (sec)')                        % add a label to the lower x axis
ylabel('xc')                                % add a label to the lower y axis
grid                                        % add grid to lower plot
```



As expected the controller states are different from the ones previously plotted, and the overall closed loop response is somewhat slower.

Simulate Open-Loop Response

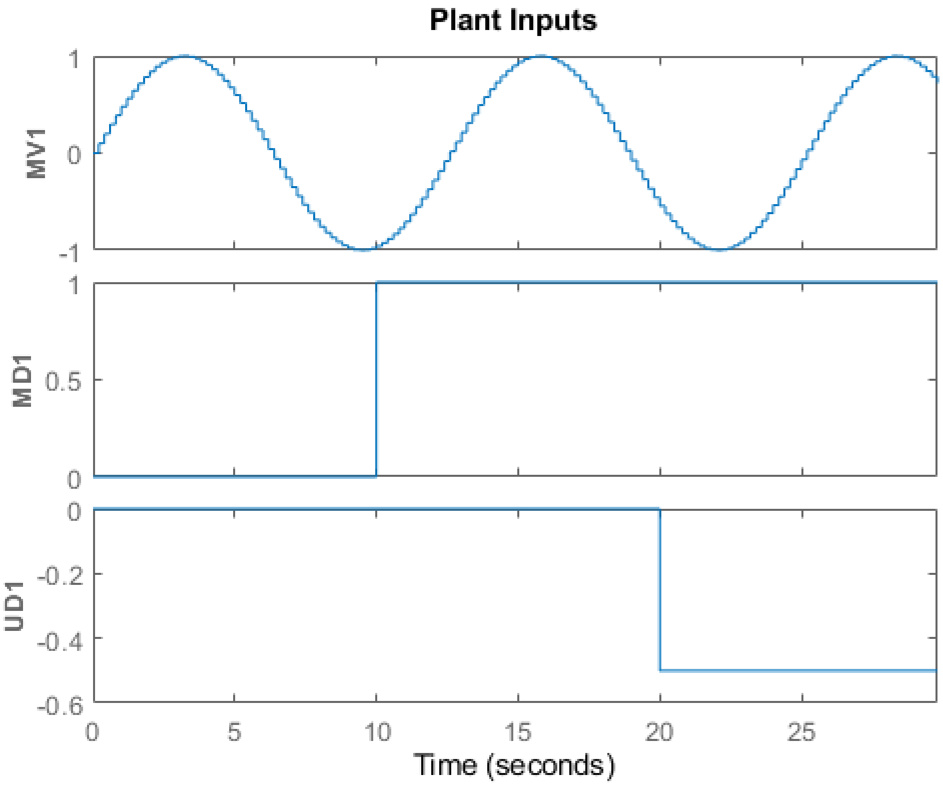
Test the behavior of the plant and controller in open-loop, using the `sim` command. Set the `OpenLoop` flag to `on`, and provide the sequence of manipulated variables to excite the system.

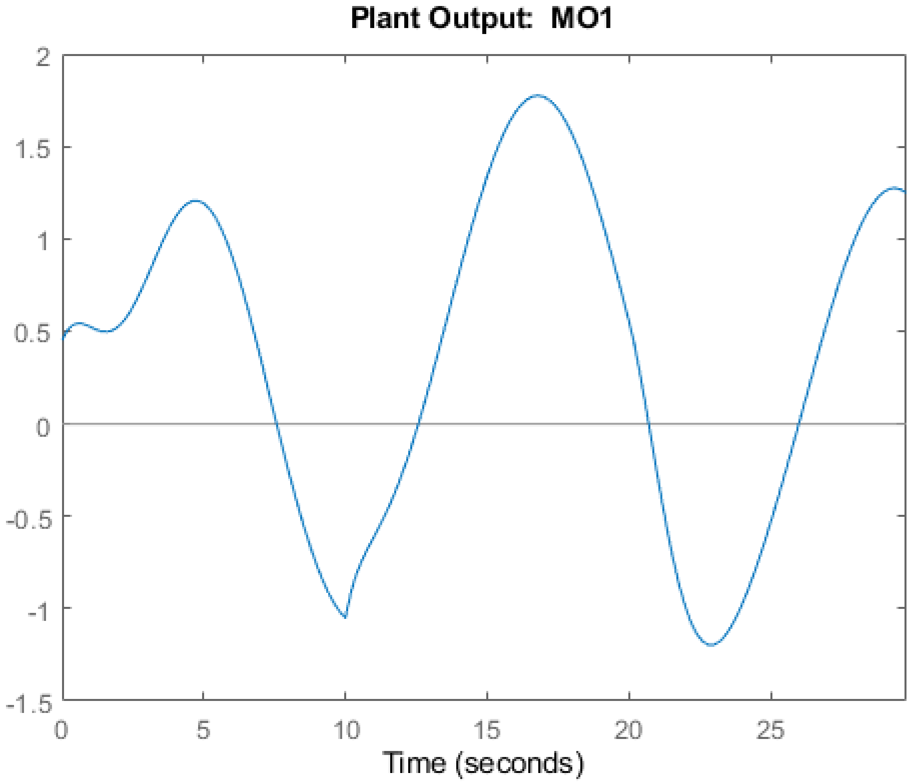
```
SimOptions.OpenLoop = 'on';                  % set open loop option
SimOptions.MVSignal = sin((0:Nf-1)'/10);     % define the manipulated variable signal
```

Simulate the open loop system containing the true plant (previously specified in `SimOptions.Model`) followed by the controller. As the reference signal will be ignored, you can avoid specifying it.

```
sim(mpcobj,Nf,[],v,SimOptions)               % simulate the open loop system
```

```
-->Converting model to discrete time.
```


Plant Inputs
MV1
1
0
-1
MD1
1
0.5
0
UD1
0
-0.2
-0.4
-0.6
0
5
10
15
20
25
Time (seconds)



Simulate Controller in Closed Loop Using mpcmove

In the more general case of a closed loop in which the controller is applied to a nonlinear or time varying plant, constraints or weights can change at run time, or the disturbance and reference signals are hard to specify completely ahead of time, you can use the `mpcmove` function at each step in a for loop to simulate the MPC controller in closed loop. If your plant is continuous-time, you will need to convert it to discrete-time (using any method) to simulate it with a for loop.

First, obtain the discrete-time state-space matrices of the plant, and define simulation time and initial states for plant and controller.

```
[A,B,C,D] = ssdata(plantDSS);       % discrete-time plant plant ss matrices
Tstop = 30;                         % Simulation time
x = [0 0 0 0 0]';                   % Initial state of the plant
xmpc = mpcstate(mpcobj);            % Initial state of the MPC controller
r = 1;                              % Output reference signal
```

Display the initial state of the controller. Note that this is an `mpcstate` object (not a simple structure) and contains the controller states only at the current time (not the whole history up to the current time, as the structure returned by sim). It also contains the estimator covariance matrix. At each simulation step, you need to update `xmpc` with the new controller states.

```
xmpc                                % controller states
```

```
MPCSTATE object with fields
          Plant: [0 0 0 0 0]
    Disturbance: 0
```

```
          Noise: [1x0 double]
       LastMove: 0
     Covariance: []
```

Define workspace arrays YY and UU to store the closed-loop trajectories so that they can be plotted after the simulation.

```
YY=[];
UU=[];
```

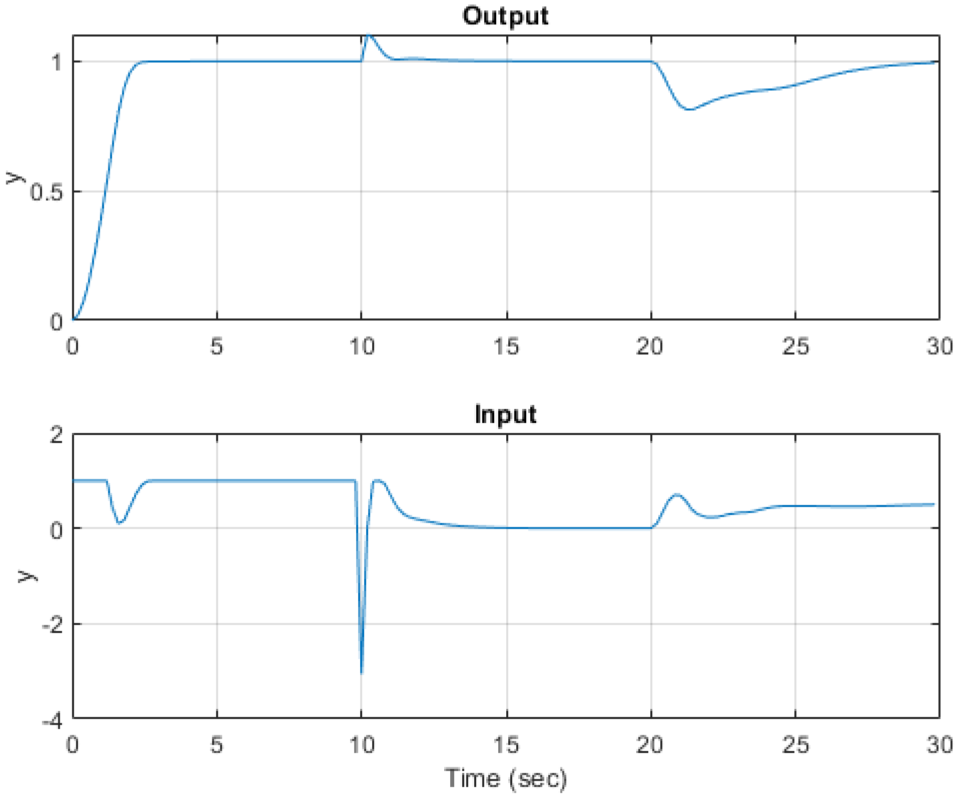
Run the simulation loop

```
for k=0:round(Tstop/Ts)-1

    % Define measured disturbance signal v(k). You might specify a more
    % complex dependence on time or previous states here.
    v = 0;
    if k*Ts>=10         % raising to 1 after 10 seconds
        v = 1;
    end

    % Define unmeasured disturbance signal d(k). You might specify a more
    % complex dependence on time or previous states here.
    d = 0;
    if k*Ts>=20          % falling to -0.5 after 20 seconds
       d = -0.5;
    end

    % Plant equations: current output
    % If you have a more complex plant output behavior (including, for example,
    % model mismatch or nonlinearities) you might want to simulate it here.
    % Note that there cannot be any direct feedthrough between u and y.
    y = C*x + D(:,2)*v + D(:,3)*d;   % calculate current output (D(:,1)=0)
    YY = [YY,y];                     % store current output

    % Compute the MPC action (u) and update the controller states (xmpc) using mpcmove
    % Note that xmpc is updated by mpcmove even though it is an input argument!
    u = mpcmove(mpcobj,xmpc,y,r,v);     % xmpc,y,r,v are values at current step k

    % Plant equations: state update
    % You might want to simulate a more complex plant state behavior here.
    x = A*x + B(:,1)*u + B(:,2)*v + B(:,3)*d;   % calculate next state and update current state
    UU = [UU,u];                                % store current input
end
```

Plot the results.

```
figure                                      % create figure
subplot(2,1,1)                              % create upper subplot axis
plot(0:Ts:Tstop-Ts,YY)                      % plot YY versus time
ylabel('y')                                 % add a label to the upper y axis
grid                                        % add grid to upper plot
title('Output')                             % add title to upper plot
subplot(2,1,2)                              % create lower subplot axis
plot(0:Ts:Tstop-Ts,UU)                      % plot UU versus time
ylabel('y')                                 % add a label to the lower y axis
xlabel('Time (sec)')                        % add a label to the lower x axis
```

```
grid                                         % add grid to lower plot
title('Input')                               % add title to lower plot
```



If at any time during the simulation you want to check the optimal predicted trajectories from that point, they can be returned by `mpcmove` as well. Assume you start from the current state (`x` and `xmpc`), while the reference set-point changes to 0.5 and that the disturbance is gone (and that both signals remain constant during the prediction horizon).

```
r = 0.5;                                     % reference
v = 0;                                       % disturbance
[~,info] = mpcmove(mpcobj,xmpc,y,r,v);       % solve over prediction horizon
```

Display the info variable.

```
info
```

```
info = 

  struct with fields:

          Uopt: [11x1 double]
          Yopt: [11x1 double]
          Xopt: [11x6 double]
          Topt: [11x1 double]
         Slack: 0
    Iterations: 1
        QPCode: 'feasible'
```

```
          Cost: 0.1399
```

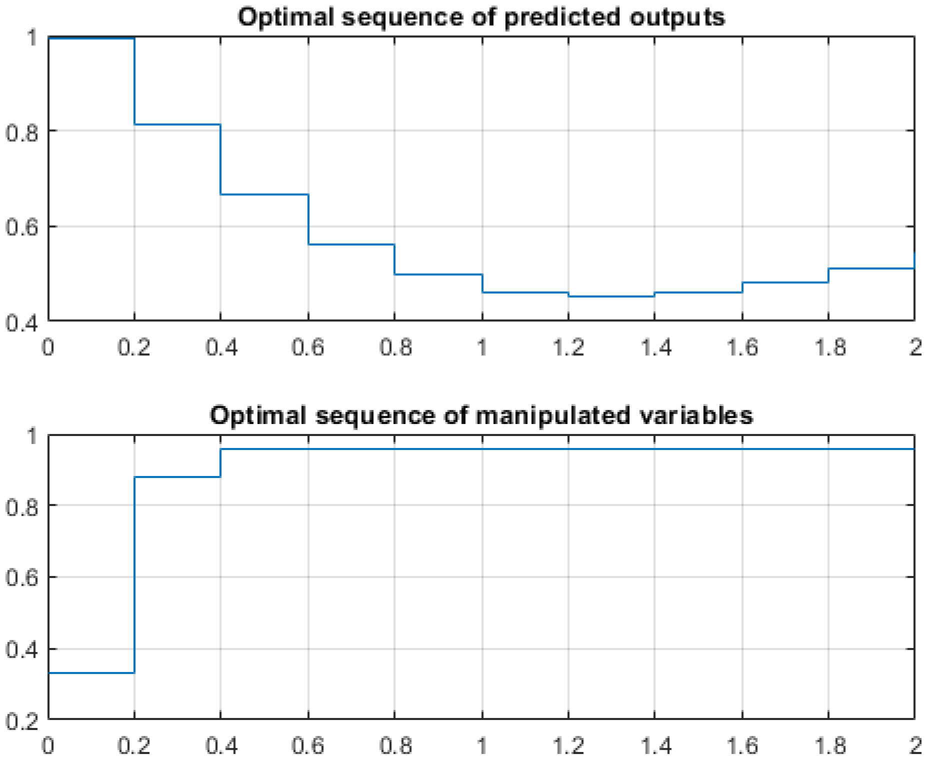
info is a structure containing the predicted optimal sequences of manipulated variables, plant states, and outputs over the prediction horizon. This sequence is calculated, together with the optimal move, by solving a quadratic optimization problem to minimize the cost function. The plant states and outputs in info result from applying the optimal manipulated variable sequence directly to mpcobj.Model.Plant, in an open-loop fashion. Therefore, this open-loop optimization process is therefore not equivalent to simulating the closed loop consisting of plant, estimator and controller using either the sim command or mpcmove iteratively in a for loop.

Extract the predicted optimal trajectories.

```
topt = info.Topt;                   % time
yopt = info.Yopt;                   % predicted optimal plant model outputs
uopt = info.Uopt;                   % predicted optimal mv sequence
```

Plot the trajectories using stairstep plots. The stairstep plots are used because a normal plot would simply join each calculated point with the following one using a direct straight line, while the optimal discrete-time signals jump instantly at each step and stay constant until the next step. This difference is especially visible since there are only 10 intervals to be plotted for the plant output (and only 3 for the manipulated variable).

```
figure                                                 % create new figure
subplot(2,1,1)                                         % create upper subplot
stairs(topt,yopt)                                      % plot yopt in a stairstep graph
title('Optimal sequence of predicted outputs')         % add title to upper subplot
grid                                                   % add grid to upper subplot
subplot(2,1,2)                                         % create lower subplot
stairs(topt,uopt)                                      % plot uopt in a stairstep graph
title('Optimal sequence of manipulated variables')     % add title to upper subplot
grid                                                   % add grid to upper subplot
```



Linear Representation of MPC Controller

When the constraints are not active, the MPC controller behaves like a linear controller. Note that for a finite-time unconstrained Linear Quadratic Regulator problem with a finite non-receding horizon, the value function is time-dependent, which causes the optimal feedback gain to be time varying. In contrast, in MPC the horizon has a constant lenght, because it is always receding, resulting in a time-invariant value function, and consequently in a time-invariant optimal feedback gain.

You can get the state-space form of the MPC controller.

```
LTI = ss(mpcobj,'rv');                  % get state space representation
```

Get the state-space matrices to simulate the linearized controller.

```
[AL,BL,CL,DL] = ssdata(LTI);            % get state space matrices
```

Initialize variables for a closed loop simulation of both the original MPC controller without constraints and the linearized controller

```
mpcobj.MV = [];             % remove input constraints
mpcobj.OV = [];             % remove output constraints

Tstop = 5;                  % set simulation time
y = 0;                      % set nitial measured output
r = 1;                      % set output reference set-point (constant)
u = 0;                      % set previous (initial) input command
```

```
x = [0 0 0 0 0]';           % set initial state of plant
xmpc = mpcstate(mpcobj);    % set initial state of unconstrained MPC controller
xL = zeros(size(BL,1),1);   % set initial state of linearized MPC controller

YY = [];                    % define workspace array to store plant outputs
```

```
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
```

Simulate both controllers in a closed loop with the same plant model

```
for k = 0:round(Tstop/Ts)-1

    YY = [YY,y];            % store current output for plotting purposes

    % Define measured disturbance signal v(k).
    v = 0;
    if k*Ts>=10
        v = 1;              % raising to 1 after 10 seconds
    end

    % Define unmeasured disturbance signal d(k).
    d = 0;
    if k*Ts>=20
        d = -0.5;           % falling to -0.5 after 20 seconds
    end

    % Compute the control actions of both (unconstrained) MPC and linearized MPC
    uMPC = mpcmove(mpcobj,xmpc,y,r,v);      % unconstrained MPC (this also updates xmpc)
    u = CL*xL+DL*[y;r;v];                   % unconstrained linearized MPC

    % Compare the two control actions
    dispStr(k+1) = {sprintf('t=%5.2f, u=%7.4f (provided by LTI), u=%7.4f (provided by MPCOBJ)',k*

    % Update state of unconstrained linearized MPC controller
    xL = AL*xL + BL*[y;r;v];

    % Update plant state
    x = A*x + B(:,1)*u + B(:,2)*v + B(:,3)*d;

    % Calculate plant output
    y = C*x + D(:,1)*u + D(:,2)*v + D(:,3)*d;       % D(:,1)=0

end
```

Display the character arrays containing the control actions

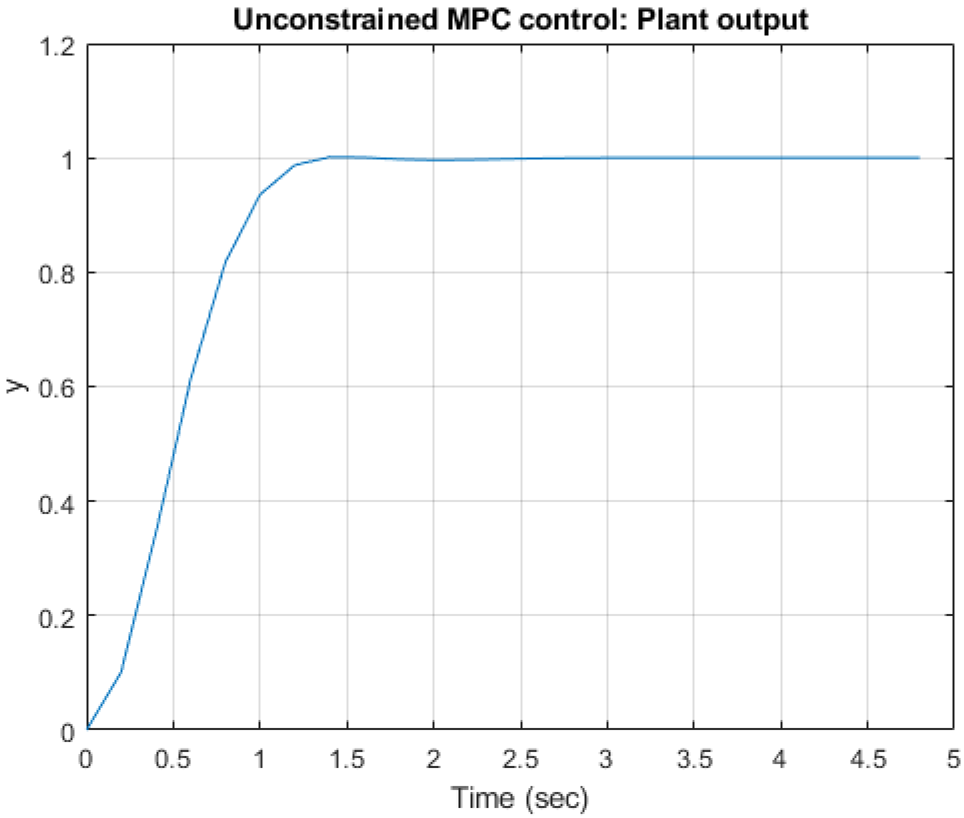
```
for k=0:round(Tstop/Ts)-1
    disp(dispStr{k+1});             % display each string as k increases
end
```

```
t= 0.00, u= 5.2478 (provided by LTI), u= 5.2478 (provided by MPCOBJ)
t= 0.20, u= 3.0134 (provided by LTI), u= 3.0134 (provided by MPCOBJ)
t= 0.40, u= 0.2281 (provided by LTI), u= 0.2281 (provided by MPCOBJ)
t= 0.60, u=-0.9952 (provided by LTI), u=-0.9952 (provided by MPCOBJ)
t= 0.80, u=-0.8749 (provided by LTI), u=-0.8749 (provided by MPCOBJ)
t= 1.00, u=-0.2022 (provided by LTI), u=-0.2022 (provided by MPCOBJ)
t= 1.20, u= 0.4459 (provided by LTI), u= 0.4459 (provided by MPCOBJ)
t= 1.40, u= 0.8489 (provided by LTI), u= 0.8489 (provided by MPCOBJ)
```

```
t= 1.60, u= 1.0192 (provided by LTI), u= 1.0192 (provided by MPCOBJ)
t= 1.80, u= 1.0511 (provided by LTI), u= 1.0511 (provided by MPCOBJ)
t= 2.00, u= 1.0304 (provided by LTI), u= 1.0304 (provided by MPCOBJ)
t= 2.20, u= 1.0053 (provided by LTI), u= 1.0053 (provided by MPCOBJ)
t= 2.40, u= 0.9920 (provided by LTI), u= 0.9920 (provided by MPCOBJ)
t= 2.60, u= 0.9896 (provided by LTI), u= 0.9896 (provided by MPCOBJ)
t= 2.80, u= 0.9925 (provided by LTI), u= 0.9925 (provided by MPCOBJ)
t= 3.00, u= 0.9964 (provided by LTI), u= 0.9964 (provided by MPCOBJ)
t= 3.20, u= 0.9990 (provided by LTI), u= 0.9990 (provided by MPCOBJ)
t= 3.40, u= 1.0002 (provided by LTI), u= 1.0002 (provided by MPCOBJ)
t= 3.60, u= 1.0004 (provided by LTI), u= 1.0004 (provided by MPCOBJ)
t= 3.80, u= 1.0003 (provided by LTI), u= 1.0003 (provided by MPCOBJ)
t= 4.00, u= 1.0001 (provided by LTI), u= 1.0001 (provided by MPCOBJ)
t= 4.20, u= 1.0000 (provided by LTI), u= 1.0000 (provided by MPCOBJ)
t= 4.40, u= 0.9999 (provided by LTI), u= 0.9999 (provided by MPCOBJ)
t= 4.60, u= 1.0000 (provided by LTI), u= 1.0000 (provided by MPCOBJ)
t= 4.80, u= 1.0000 (provided by LTI), u= 1.0000 (provided by MPCOBJ)
```

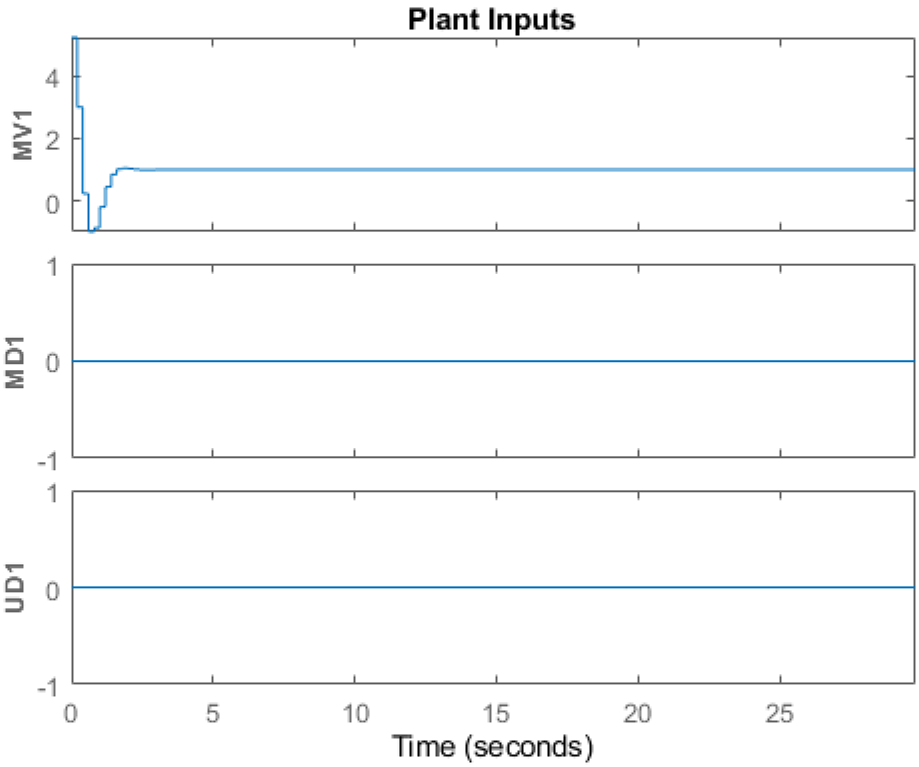
Plot the results.

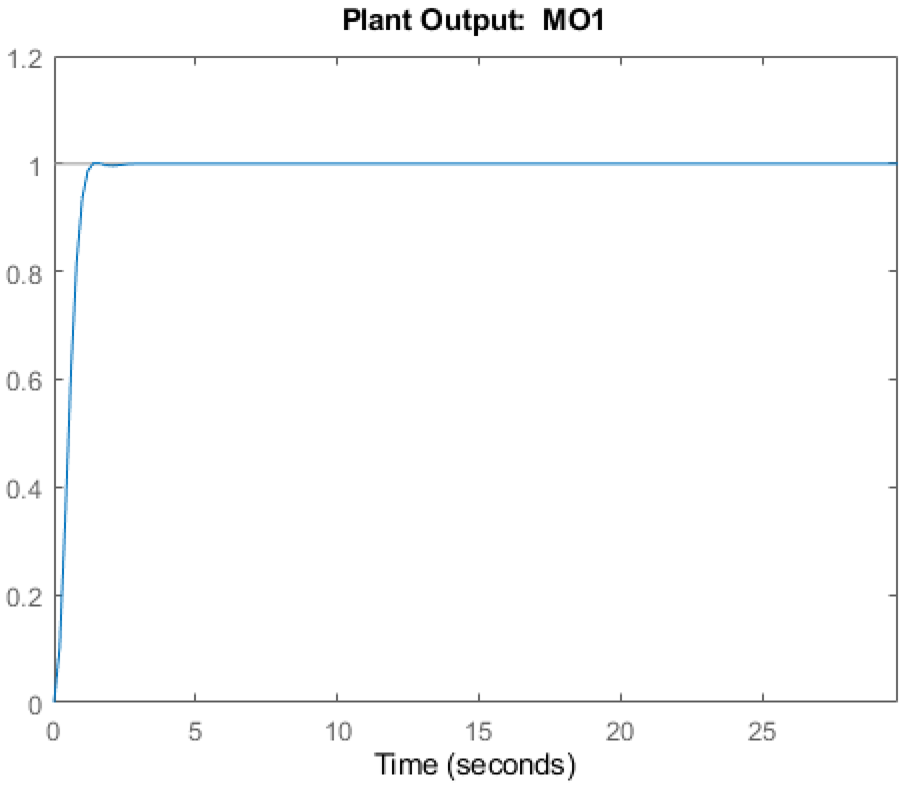
```
figure                                              % create figure
plot(0:Ts:Tstop-Ts,YY)                              % plot plant outputs
grid                                                % add grid
title('Unconstrained MPC control: Plant output')    % add title
xlabel('Time (sec)')                                % add label to x axis
ylabel('y')                                         % add label to y axis
```



Running a closed-loop simulation in which all controller constraints are turned off is easier using `sim`, as you just need to specify 'off' in the `Constraint` field of the related `mpcsimopt` simulation option object.

```
SimOptions = mpcsimopt;                       % create simulation options object
SimOptions.Constraints = 'off';               % remove all MPC constraints
SimOptions.UnmeasuredDisturbance = d;         % unmeasured input disturbance
sim(mpcobj,Nf,r,v,SimOptions);                % run closed loop simulation
```





Simulate Using Simulink®

Simulink is a graphical block diagram environment for multidomain system simulation. You can connect blocks representing dynamical systems (in this case the plant and the MPC controller) and simulate the closed loop. Besides the fact that the system and its interconnections are represented visually, one key difference is that Simulink can use a wider range of integration algorithms to solve the underlying system differential equations. Simulink can also automatically generate C (or PLC) code and deploy it for real-time applications.

```
% To run this part of the example, Simulink(R) is required. Check that
% Simulink is installed, otherwise display a message and return
if ~mpcchecktoolboxinstalled('simulink')    % if Simulink not installed
    disp('Simulink(R) is required to run this part of the example.')
    return                                  % end execution here
end
```

Recreate the original mpc object with the original constraint and the original default estimator, so you can easily compare results.

```
mpcobj = mpc(plantDSS,Ts,10,3);
mpcobj.MV = struct('Min',0,'Max',1,'RateMin',-10,'RateMax',10);
mpcobj.Model.Disturbance = tf(sqrt(1000),[1 0]);
```

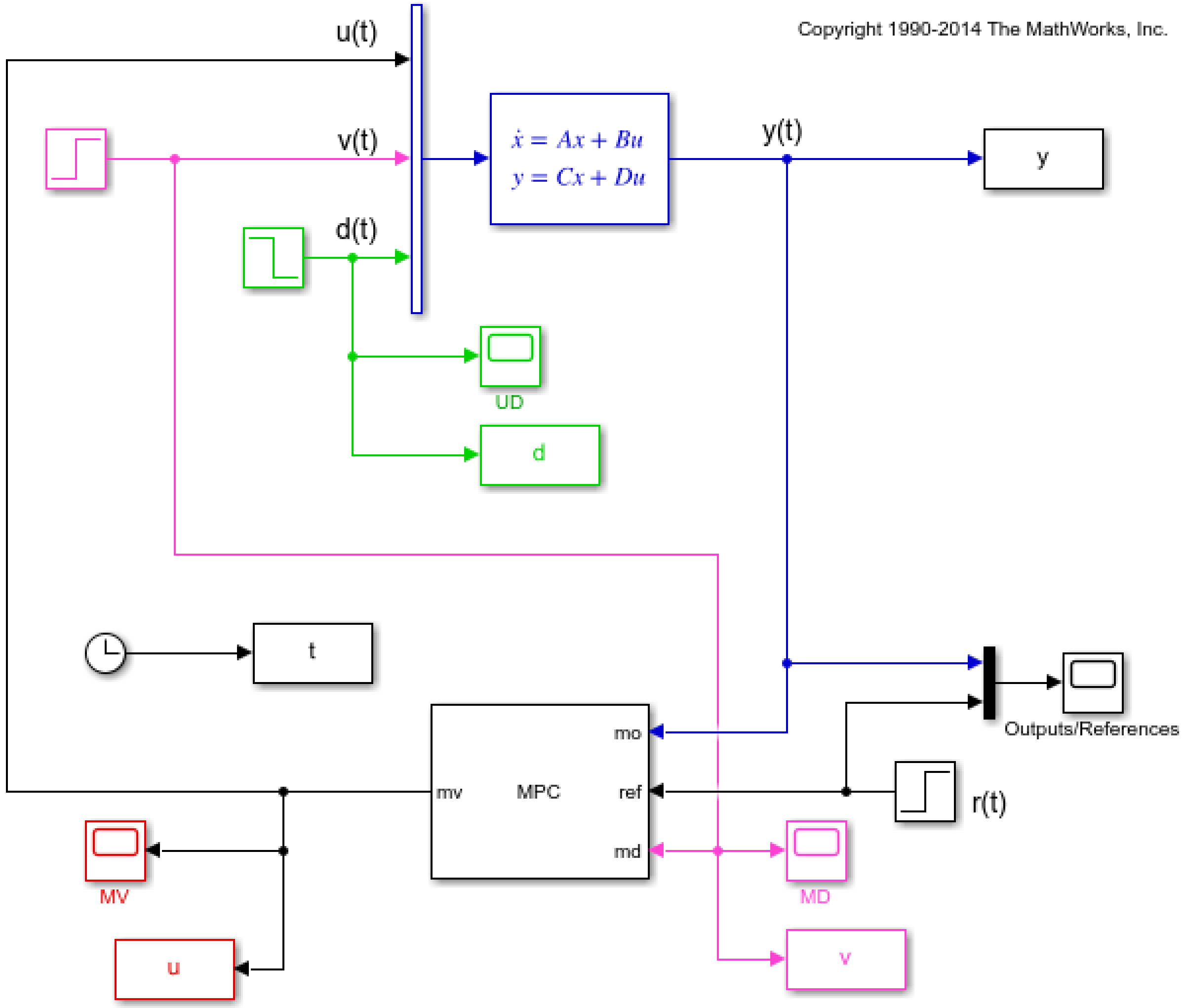
```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming default 0.00000
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming default 0.1
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1.00000.
```

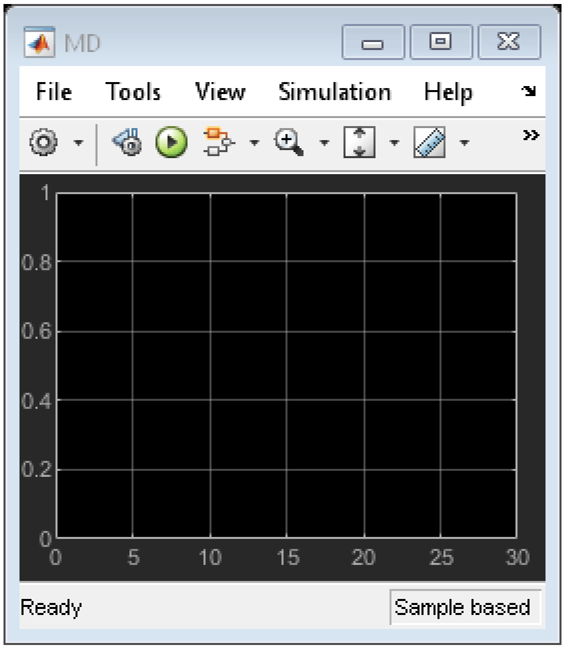
Extract the state-space matrices of the (continuous-time) plant to be controlled, since they are needed by the Simulink model to be opened.

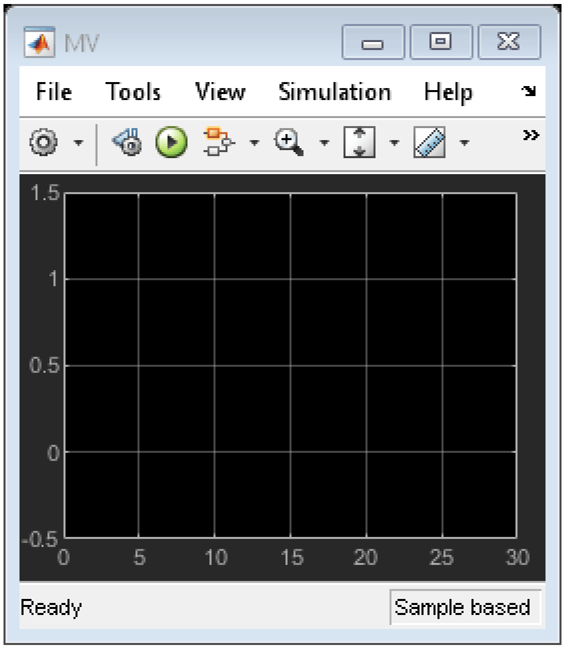
```
[A,B,C,D] = ssdata(plantCSS);       % get state-space realization
```

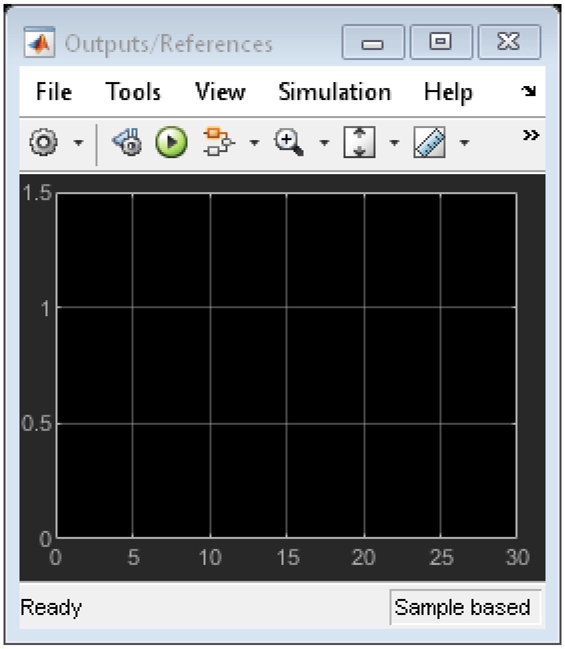
Open the pre-existing Simulink model for the closed-loop simulation. The plant model is implemented with a continuous state space block, and the ode45 integration algorithm is used to calculate its continuous time response.

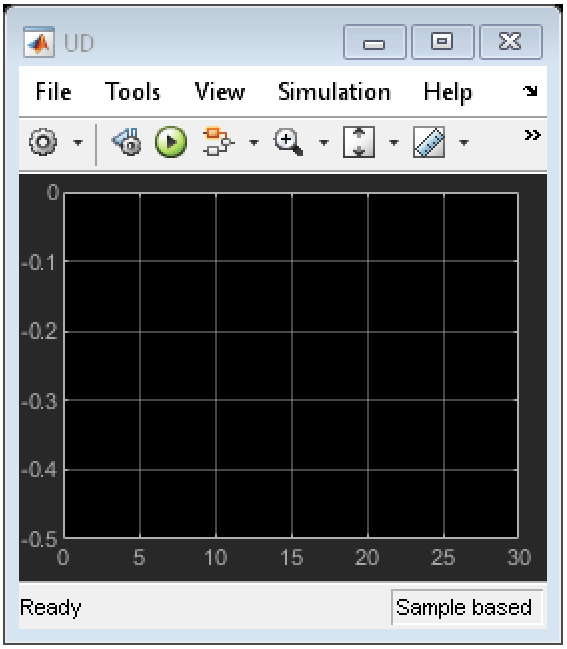
```
%The block inputs are u(t), v(t) and d(t) representing the manipulated
% variable, measured input disturbance, and unmeasured input disturbance,
% respectively, while y(t) is the measured output. The block parameters
% are the matrices forming the state space realization of the continuous-time
% plant, and the initial conditions for the 5 states.
% The MPC controller is implemented with an MPC controller block, which
% has the workspace mpc object |mpcobj| as parameter, the manipulated
% variable as output, and then the measured plant output, reference signal,
% and measured plant input disturbance as inputs, respectively.
% The four Scopes blocks plot the five loop signals, which are also saved
% (except the reference signal) by four To-Workspace blocks.
open_system('mpc_miso')
```


Copyright 1990-2014 The MathWorks, Inc.
u(t)
v(t)
d(t)
$\dot{x} = Ax + Bu$
$y = Cx + Du$
y(t)
y
UD
d
t
Outputs/References
mo
MPC
mv
ref
md
r(t)
MV
MD
u
v


MD
File Tools View Simulation Help
1
0.8
0.6
0.4
0.2
0
0 5 10 15 20 25 30
Ready
Sample based


MV
File Tools View Simulation Help
1.5
1
0.5
0
-0.5
0 5 10 15 20 25 30
Ready
Sample based

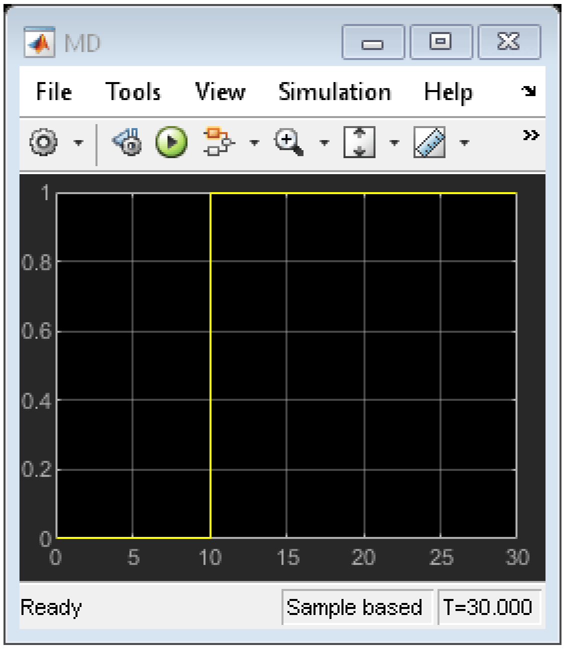


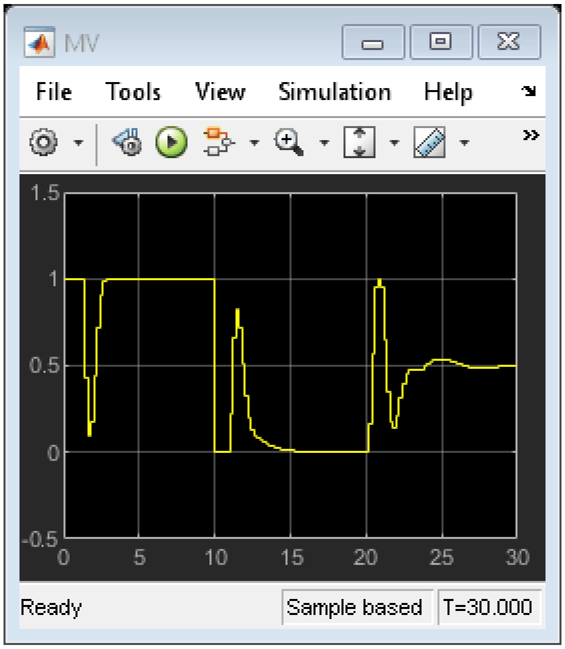


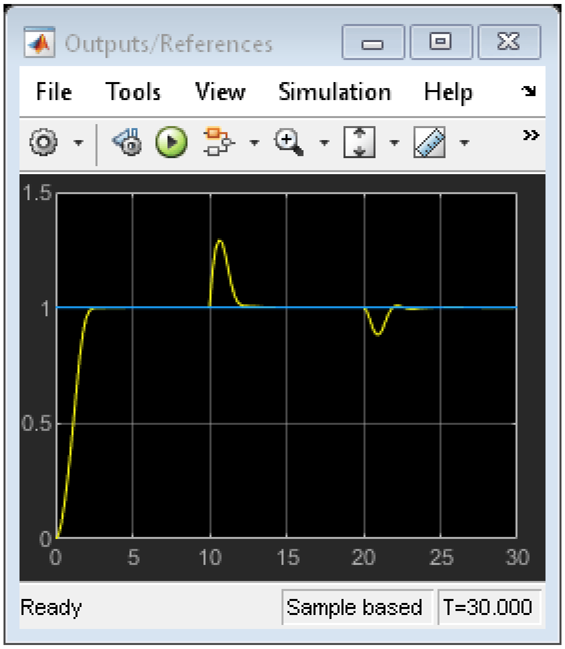
Simulate the system using the simulink `sim` command. Note that this command (which simulates a Simulink model, and is equivalent to clicking the "Run" button in the model) is different from the `sim` command provided by the mpc toolbox (which instead simulates an MPC controller in a loop with an LTI plant).

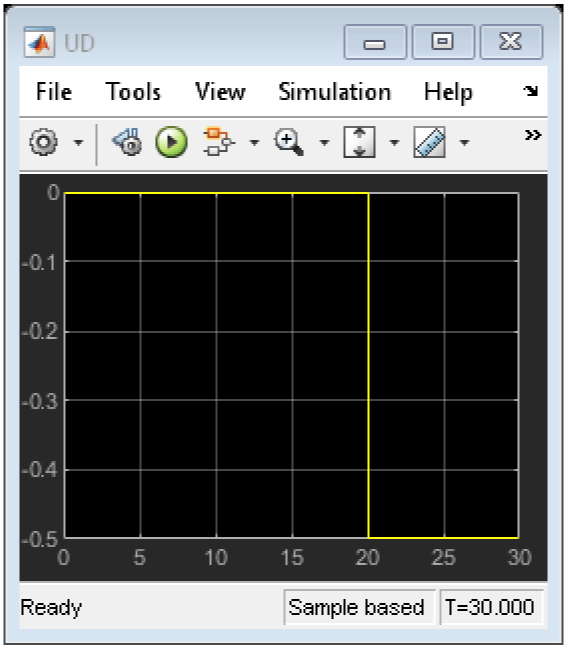
```
sim('mpc_miso')
```

```
   Assuming no disturbance added to measured output channel #1.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measure
```


MD
File
Tools
View
Simulation
Help
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
Ready
Sample based
T=30.000


MV
File
Tools
View
Simulation
Help
1.5
1
0.5
0
-0.5
0
5
10
15
20
25
30
Ready
Sample based
T=30.000





The plots in the `Scopes` blocks are equivalent to the ones in figures 1,2 and 3, with minor differences due to the fact that in the first simulation (Figures 1 and 2) the unmeasured disturbance signal was zero, and that is the second simulation (Figure 3) noise was added to the plant input and output. Also note that that, while the MPC `sim` command internally discretizes any continuous plant model using ZOH, Simulink typically uses an integration algorithm (in this example ode45) to simulate the closed loop when a continuous-time block is present.

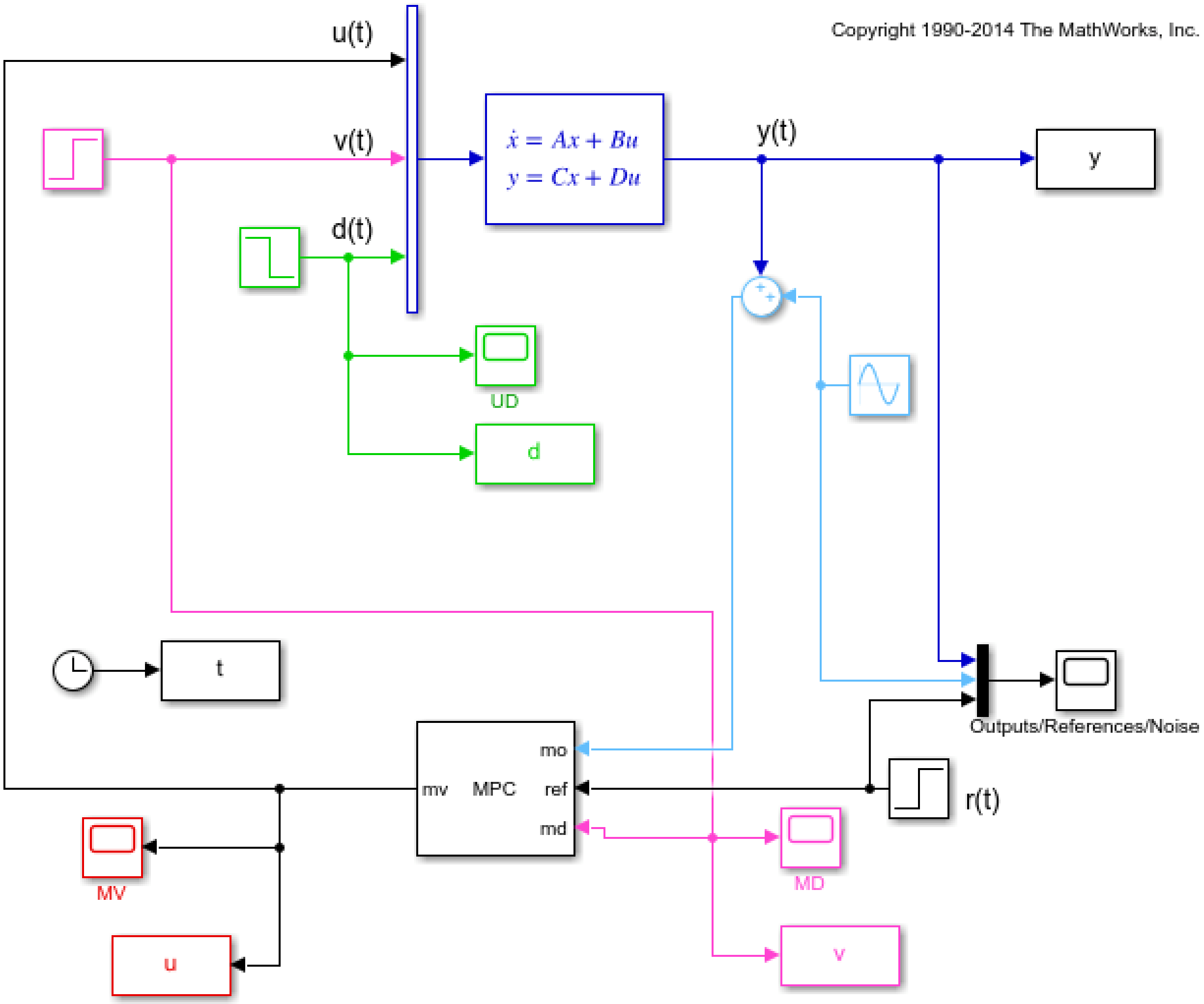
Run a simulation with sinusoidal output noise.

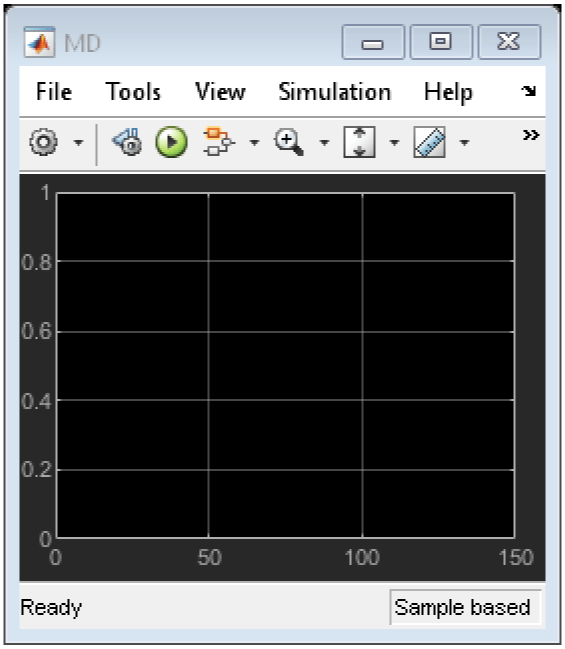
Assume output measurements are affected by a sinusoidal disturbance (a single tone sensor noise) of frequency 0.1 Hz on the measured output.

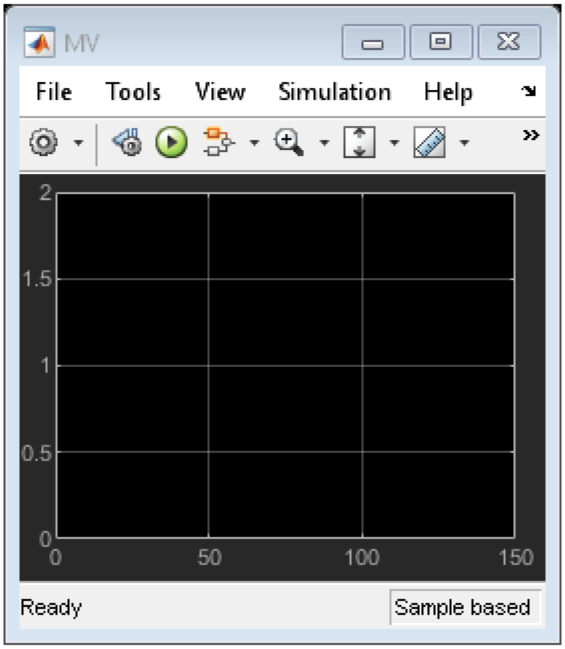
```
omega = 2*pi/10;                           % disturbance radial frequency
```

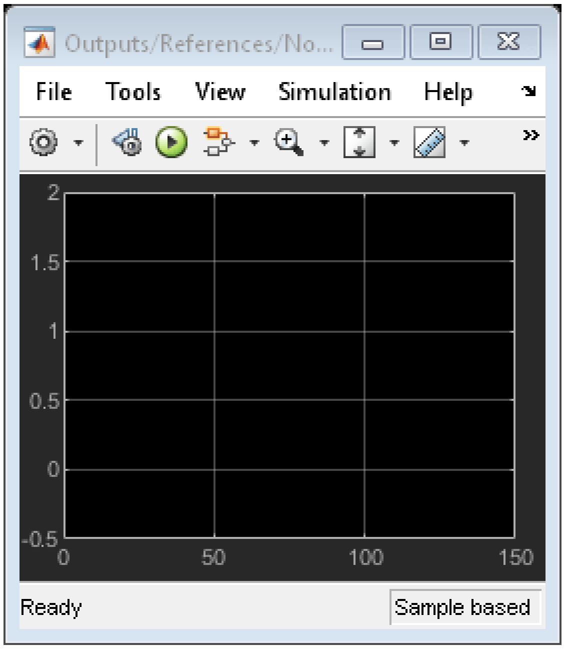
Open the `mpc_misonoise` Simulink model. It's similar to the 'mpc_miso' model except for the sinusoidal disturbance injected on the measured output. What is also different is that the simulation time is now set to 150 seconds, and that unmeasured and measured input disturbances start to act at after 60 and 120 seconds, respectively. Note that, differently from the previous simulations, the unmeasured disturbance start first. This allows the response to unmeasured disturbance, taking place from 60 to 120 seconds, to be plotted and analyzed more clearly.

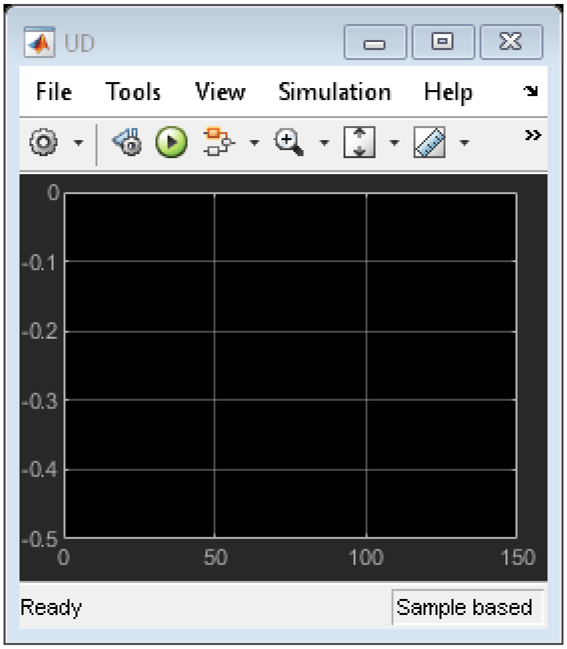
```
open_system('mpc_misonoise')               % open new Simulink model
```




MD
File
Tools
View
Simulation
Help
1
0.8
0.6
0.4
0.2
0
0
50
100
150
Ready
Sample based


MV
File
Tools
View
Simulation
Help
2
1.5
1
0.5
0
0
50
100
150
Ready
Sample based





Since this noise is expected, specifying it in a measurement noise model will help the state estimator to ignore it, thereby improving the quality of the state estimates and allowing the whole controller to better reject disturbances while tracking the output reference.

```
mpcobj.Model.Noise = 0.5*tf(omega^2,[1 0 omega^2]); % measurement noise model
```

Revise the MPC design by specifing a disturbance model on the unmeasured input as a white Gaussian noise with zero mean and variance `0.1`. This allows a better rejection of generic non constant disturbances (at the expense of a worse rejection of input constant disturbances).

```
mpcobj.Model.Disturbance = tf(0.1);                    % static gain
```

Note that when you specify a static gain as a model for the input disturbance, an output disturbance model consisting in a discretized integrator is automatically added to the controller. This helps rejecting constant (and slow varying) output disturbances.

```
getoutdist(mpcobj)
```

```
-->Assuming output disturbance added to measured output channel #1 is integrated white noise.
-->A feedthrough channel in NoiseModel was inserted to prevent problems with estimator design.

ans =

  A = 
       x1
   x1   1

  B = 
        u1
   x1  0.2

  C = 
        x1
   MO1   1

  D = 
        u1
   MO1   0

Sample time: 0.2 seconds
Discrete-time state-space model.
```

Decrease the weight on the tracking of the output variable, (thereby placing more emphasis on minimizing the rate of change of the manipulated variables).

```
mpcobj.weights = struct('MV',0,'MVRate',0.1,'OV',0.005);    % new weights
```

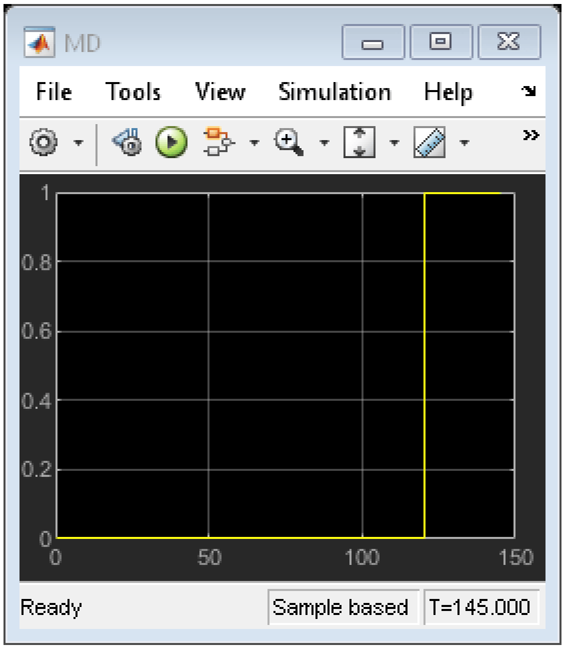
By themselves, the new weights would penalize the manipulated variable change too much, resulting in a very low bandwidth (slow) controller. This would result in the output variables lagging behind the reference for many steps. Increasing the predicion horizon to 40 allows the controller to fully take into account the cost of the output error for 40 steps instead of just 10, thereby placing back some emphasis on tracking.

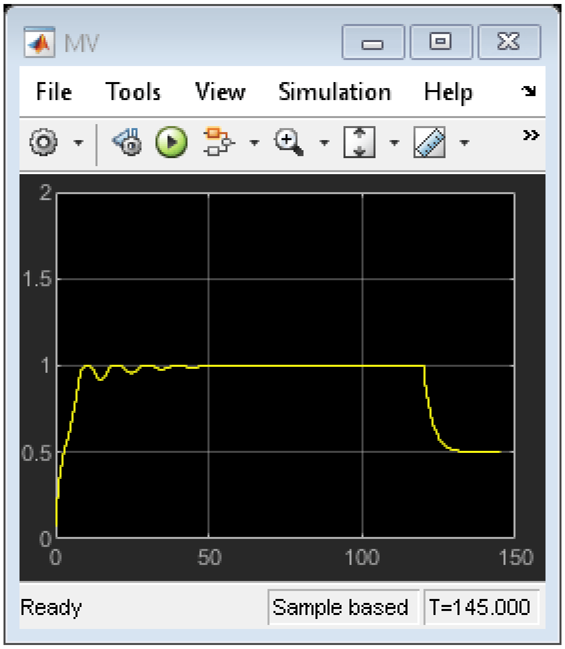
```
mpcobj.predictionhorizon = 40;                  % new prediction horizon
```

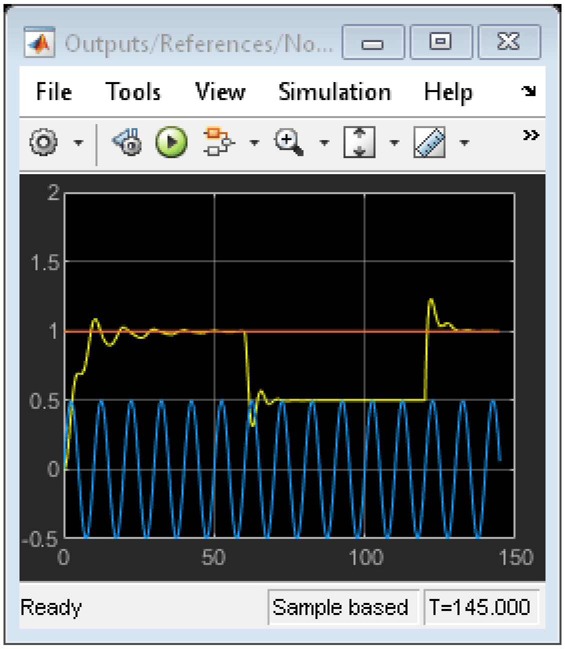
Run the simulation for 145 seconds

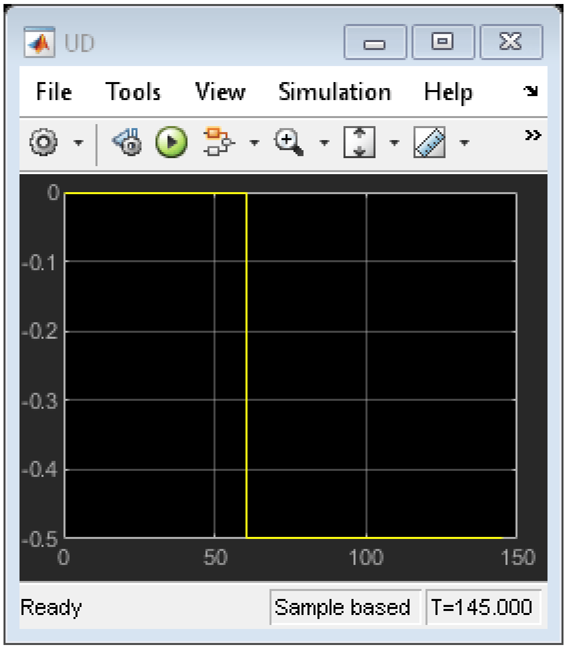
```
sim('mpc_misonoise',145)    % the second argument is the simulation duration
```

```
-->Assuming output disturbance added to measured output channel #1 is integrated white noise.
-->A feedthrough channel in NoiseModel was inserted to prevent problems with estimator design.
```


MD
File Tools View Simulation Help
1
0.8
0.6
0.4
0.2
0
0
50
100
150
Ready
Sample based
T=145.000


MV
File Tools View Simulation Help
2
1.5
1
0.5
0
0
50
100
150
Ready
Sample based
T=145.000





The kalman filter successfully learns to ignore the measurement noise after 50 seconds. The unmeasured and measured disturbances get rejected in a 10 to 20 second timespan. Finally, as expected, the manipulated variable stays in the interval between 0 and 1.

```
bdclose('all') % close all open Simulink models without saving any change
close all      % close all open figures
```

See Also

MPC Controller | **MPC Designer** | `mpc`

More About

- “Model Predictive Control of a Single-Input-Single-Output Plant” on page 3-47
- “Model Predictive Control of a Multi-Input Multi-Output Nonlinear Plant” on page 3-91

Model Predictive Control of a Multi-Input Multi-Output Nonlinear Plant

This example shows how to design a model predictive controller for a multi-input multi-output nonlinear plant and simulate the closed loop in Simulink. The plant has 3 manipulated variables and 2 measured outputs.

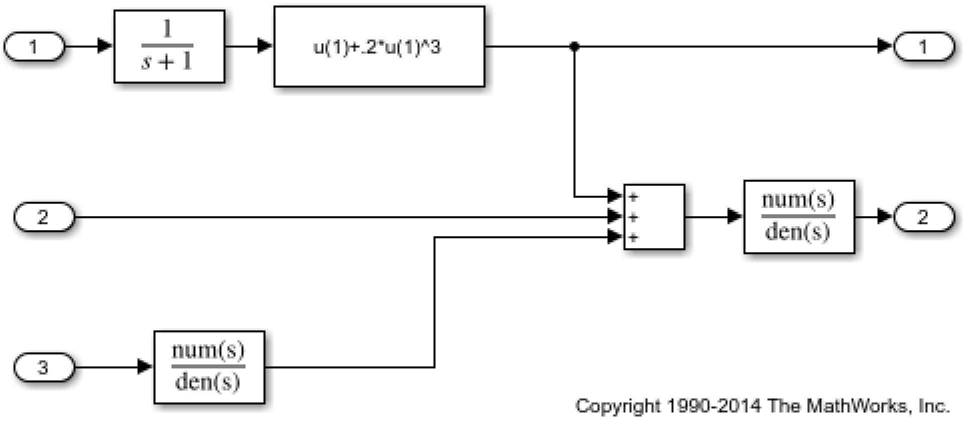
Linearize the Nonlinear Plant

To run this example, Simulink® and Simulink Control Design™ are required.

```
% Check that both Simulink and Simulink Control Design are installed,
% otherwise display a message and return
if ~mpcchecktoolboxinstalled('simulink')
    disp('Simulink(R) is required to run this example.')
    return
end
if ~mpcchecktoolboxinstalled('slcontrol')
    disp('Simulink Control Design(R) is required to run this example.')
    return
end
```

The nonlinear plant is implemented in the Simulink model `mpc_nonlinmodel`. Notice the nonlinearity `0.2*u(1)^3` from the first input to the first output.

```
open('mpc_nonlinmodel')
```



Linearize the plant at the default operating conditions (the initial states of the transfer function blocks are all zero) using the `linearize` command from the Simulink Control Design Toolbox.

```
plant = linearize('mpc_nonlinmodel');
```

Assign names to I/O variables.

```
plant.InputName = {'Mass Flow';'Heat Flow';'Pressure'};
plant.OutputName = {'Temperature';'Level'};
plant.InputUnit = {'kg/s' 'J/s' 'Pa'};
plant.OutputUnit = {'K' 'm'};
```

Note that since you have not defined any measured or unmeasured disturbance, or any an unmeasured output, when an MPC controller is created based on `plant`, by default all plant inputs are assumed to be manipulated variables and all plant outputs are assumed to be measured outputs.

Design the MPC Controller

Create the controller object with sampling period, prediction and control horizons of 0.2 sec, 5 steps, and 2 moves, respectively;

```
mpcobj = mpc(plant,0.2,5,2);
```

```
-->The "Weights.ManipulatedVariables" property of "mpc" object is empty. Assuming default 0.00000
-->The "Weights.ManipulatedVariablesRate" property of "mpc" object is empty. Assuming default 0.1
-->The "Weights.OutputVariables" property of "mpc" object is empty. Assuming default 1.00000.
```

Specify hard constraints on the manipulated variable.

```
mpcobj.MV = struct('Min',{-3;-2;-2},'Max',{3;2;2},'RateMin',{-1000;-1000;-1000});
```

Define weights on manipulated variables and output signals.

```
mpcobj.Weights = struct('MV',[0 0 0],'MVRate',[.1 .1 .1],'OV',[1 1]);
```

Display the mpc object to review its properties.

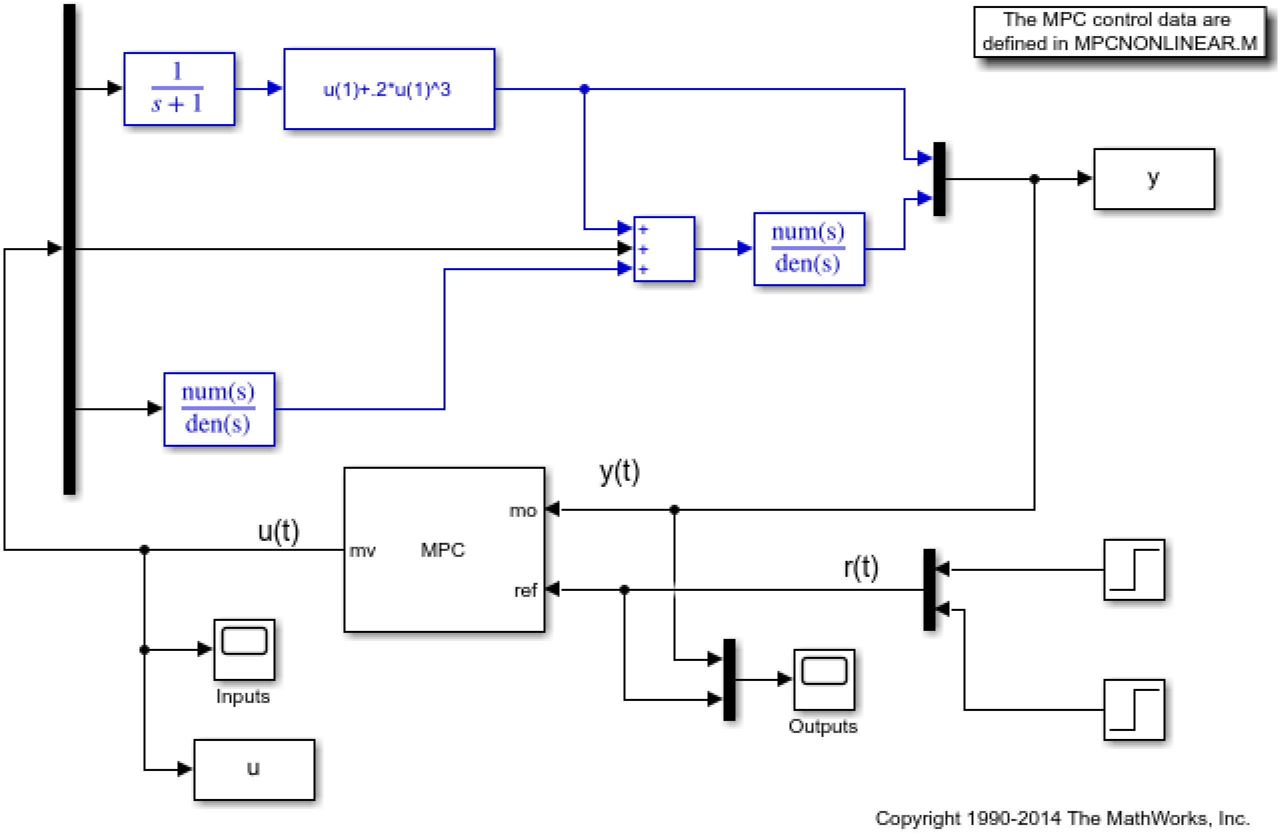
```
mpcobj
```

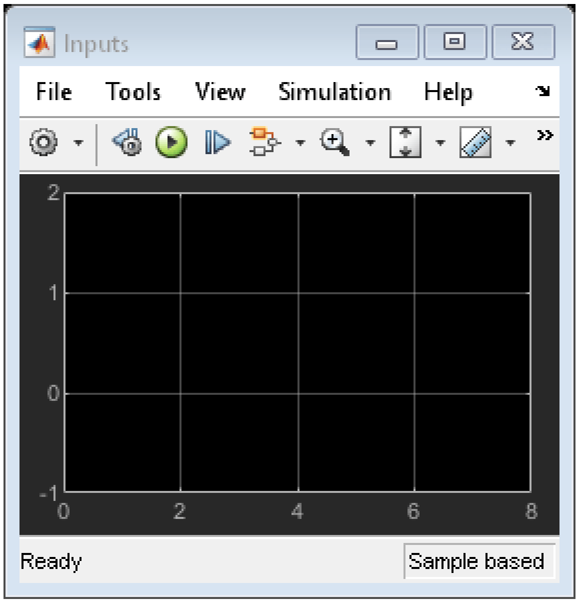
```
MPC object (created on 24-Feb-2021 00:15:05):
---------------------------------------------
Sampling time:      0.2 (seconds)
Prediction Horizon: 5
Control Horizon:    2

Plant Model:
                                      --------------
      3  manipulated variable(s)   -->|  5 states  |
                                      |            |-->  2 measured output(s)
      0  measured disturbance(s)   -->|  3 inputs  |
                                      |            |-->  0 unmeasured output(s)
      0  unmeasured disturbance(s) -->|  2 outputs |
                                      --------------
Disturbance and Noise Models:
        Output disturbance model: default (type "getoutdist(mpcobj)" for details)
         Measurement noise model: default (unity gain after scaling)

Weights:
        ManipulatedVariables: [0 0 0]
    ManipulatedVariablesRate: [0.1000 0.1000 0.1000]
             OutputVariables: [1 1]
                         ECR: 100000

State Estimation:  Default Kalman Filter (type "getEstimator(mpcobj)" for details)

Constraints:
 -3 <= Mass Flow (kg/s) <= 3, -1000 <= Mass Flow/rate (kg/s) <= Inf, Temperature (K) is unconstra
  -2 <= Heat Flow (J/s) <= 2,  -1000 <= Heat Flow/rate (J/s) <= Inf,       Level (m) is unconstra
    -2 <= Pressure (Pa) <= 2,    -1000 <= Pressure/rate (Pa) <= Inf
```

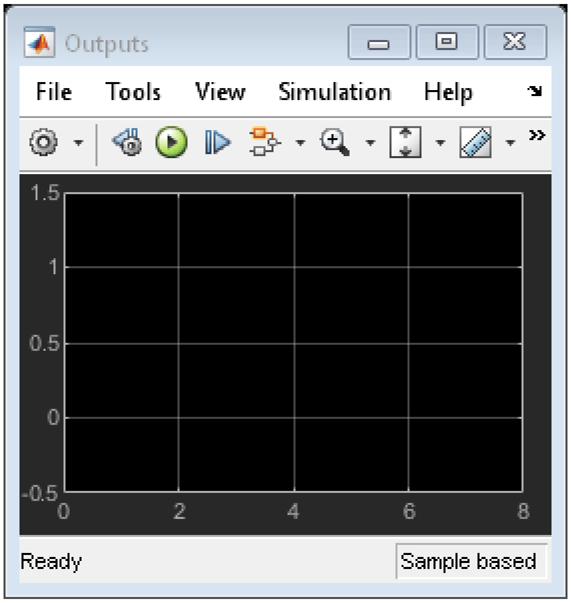
Simulate the Closed Loop Using Simulink

Open the pre-existing Simulink model for the closed-loop simulation. The plant model is identical to the one used for linearization, while the MPC controller is implemented with an MPC controller block, which has the workspace mpc object `mpcobj` as parameter. The reference for the first output is a step signal rising fron zero to one for t=0, as soon as the simulation starts. The reference for the second output

```
mdl1 = 'mpc_nonlinear';
open_system(mdl1)
```


The MPC control data are defined in MPCNONLINEAR.M
1
s + 1
u(1)+.2*u(1)^3
num(s)
den(s)
+
+
+
num(s)
den(s)
y
y(t)
mo
u(t)
mv
MPC
ref
r(t)
Inputs
Outputs
u
Copyright 1990-2014 The MathWorks, Inc.

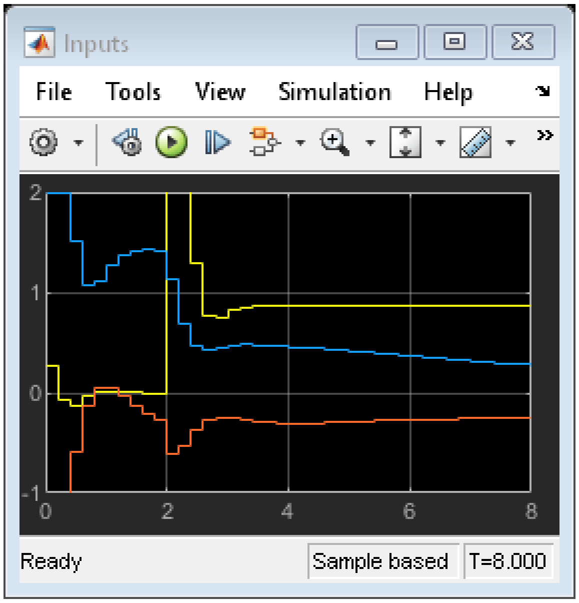


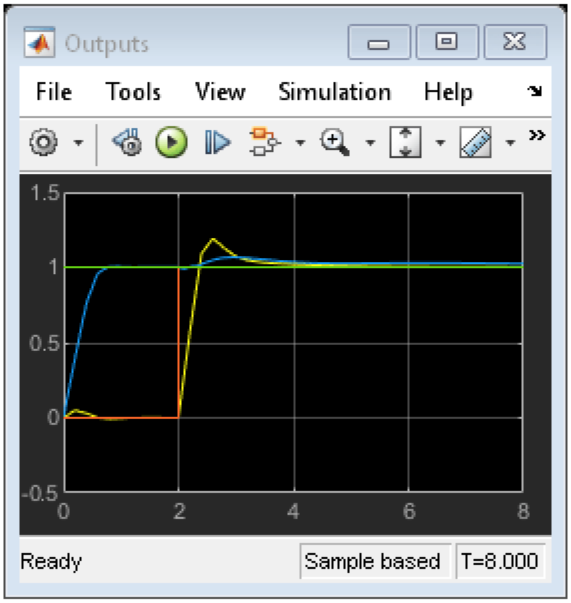


Run the closed loop simulation.

```
sim(mdl1)
```

```
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #1 is integrated white noise.
-->Assuming output disturbance added to measured output channel #2 is integrated white noise.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
```





Despite the presence of the nonlinearity, both outputs track their references well after a few seconds, while, as expected, the manipulated variables stay within the preset hard constraints.

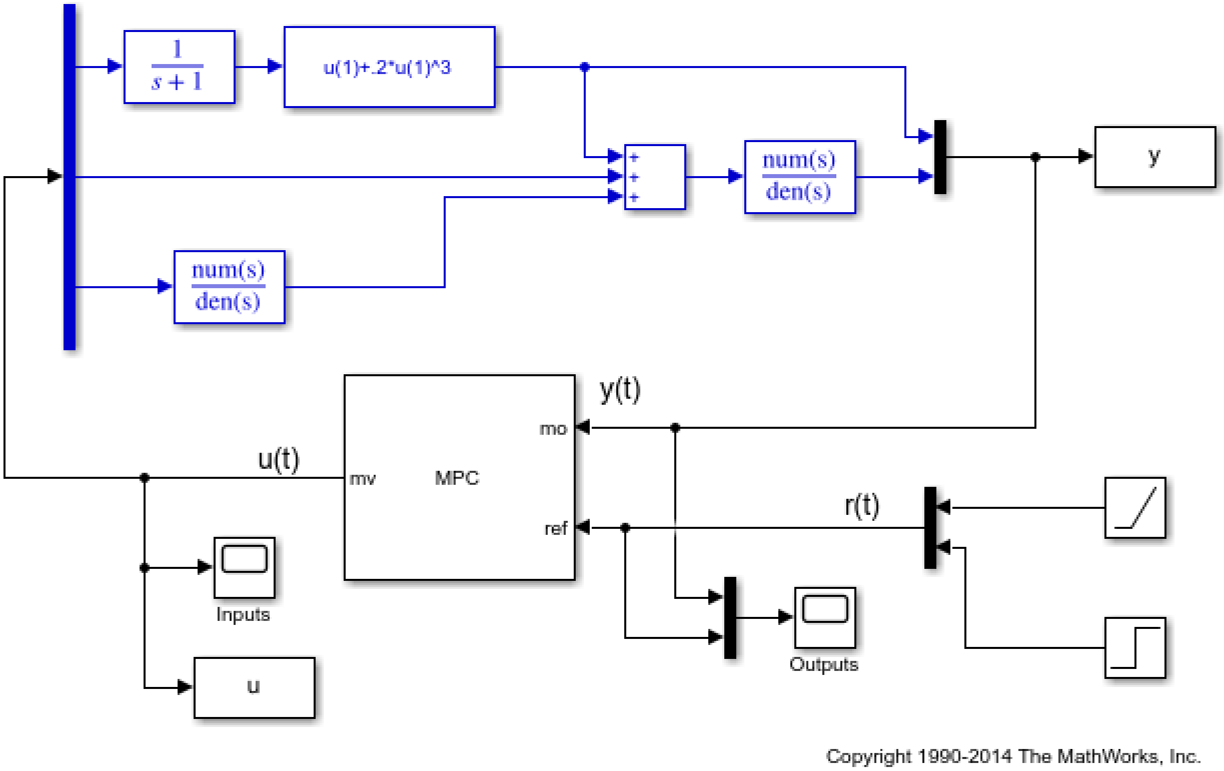
Modify MPC Design to Track Ramp Signals

In order to both track a ramp while compensating for the nonlinearity, define a disturbance model on both outputs as a triple integrator (without the nonlinearity a double integrator would suffice).

```
outdistmodel = tf({1 0;0 1},{[1 0 0 0],1;1,[1 0 0 0]});
setoutdist(mpcobj,'model',outdistmodel);
```

Open the pre-existing Simulink model for the closed-loop simulation. It is identical to the previous closed loop model, except for the fact that the reference for the first plant output is npo longer a step but a ramp signal that rises with slope of 0.2 after 3 seconds.

```
mdl2 = 'mpc_nonlinear_setoutdist';
open_system(mdl2)
```



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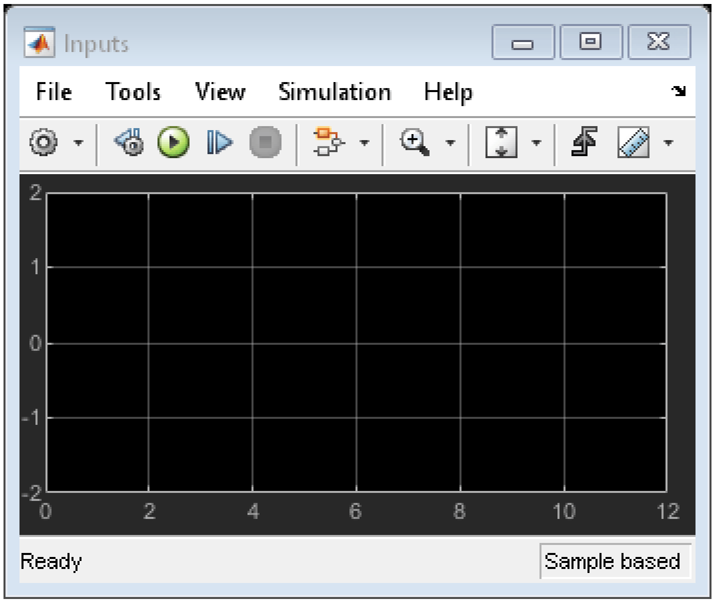
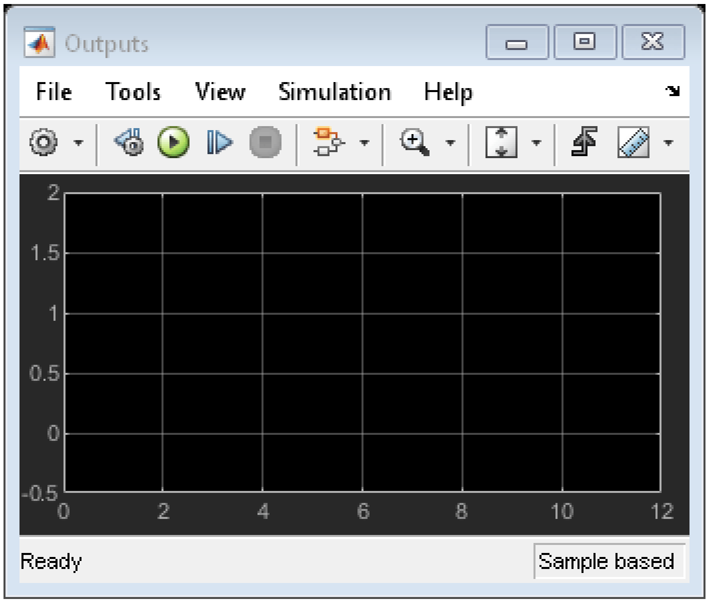
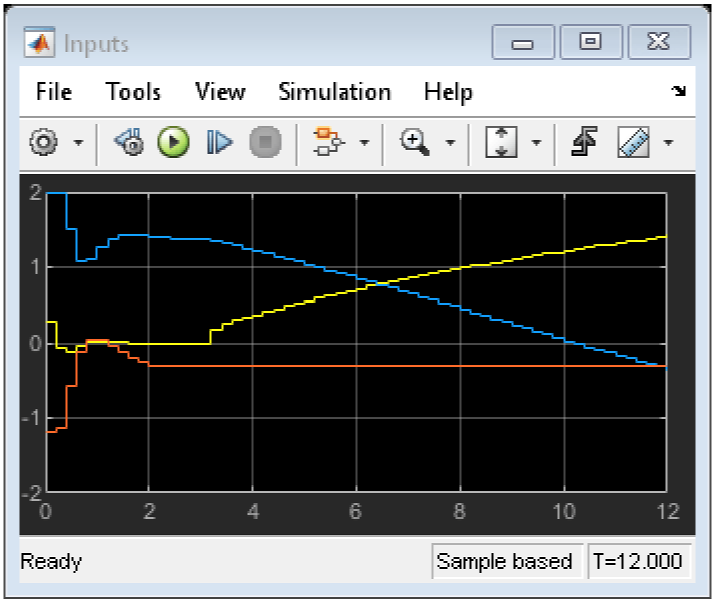


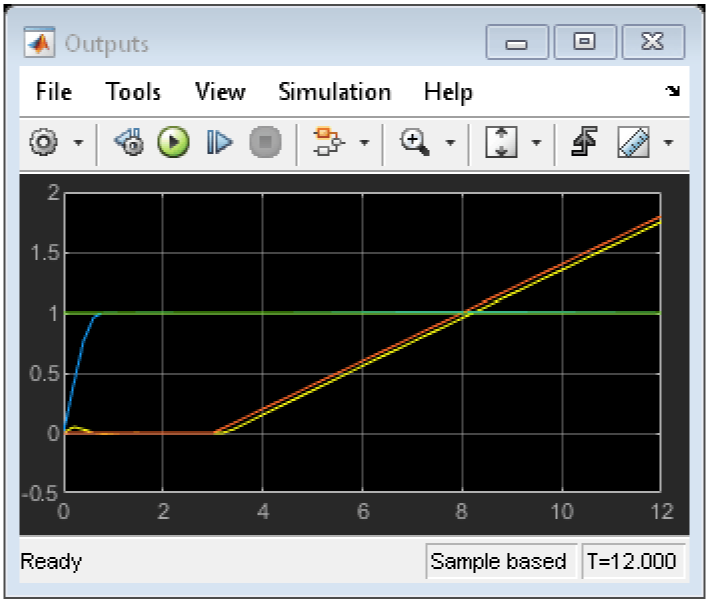


Run the closed loop simulation for 12 seconds.

```
sim(mdl2,12)
```

```
-->Converting model to discrete time.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
```





Simulate without Constraints

When the constraints are not active, the MPC controller behaves like a linear controller. Simulate two versions of an unconstrained MPC controller in closed loop to illustrate this fact.

First, remove the constraints from mcpobj.

```
mpcobj.MV = [];
```

Then reset the output disturbance model to default, (this is only done to get a simpler version of a linear MPC controller in the next step).

```
setoutdist(mpcobj,'integrators');
```

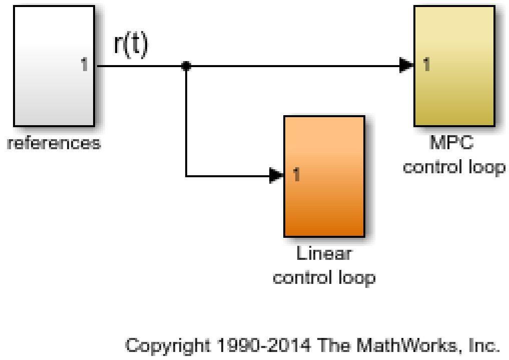
Convert the unconstrained MPC controller to a linear time invariant (LTI) state space dynamical system, having the vector [ym;r] as input, where ym is the vector of measured output signals (at a given step), and r is the vector of output references (at the same given step).

```
LTI = ss(mpcobj,'r');         % use reference as additional input signal
```

```
-->Converting model to discrete time.
-->Assuming output disturbance added to measured output channel #1 is integrated white noise.
-->Assuming output disturbance added to measured output channel #2 is integrated white noise.
-->The "Model.Noise" property of the "mpc" object is empty. Assuming white noise on each measured
```

Open the pre-existing Simulink model for the closed-loop simulation. The "references" block contains two step signals (acting after 4 and 0 seconds, respectively) that are used as a reference. The "MPC control loop" block is equivalent to the first closed loop, except for the fact that the reference signals are supplied to it as input. The "Linear control loop" block is equivalent to the "MPC control loop" block except for the fact that the controller is an LTI block having the workspace ss object LTI as parameter.

```
refs = [1;1];                  % set values for step signal references
mdl3 = 'mpc_nonlinear_ss';
open_system(mdl3)
```


r(t)
1
references
1
MPC
control loop
1
Linear
control loop
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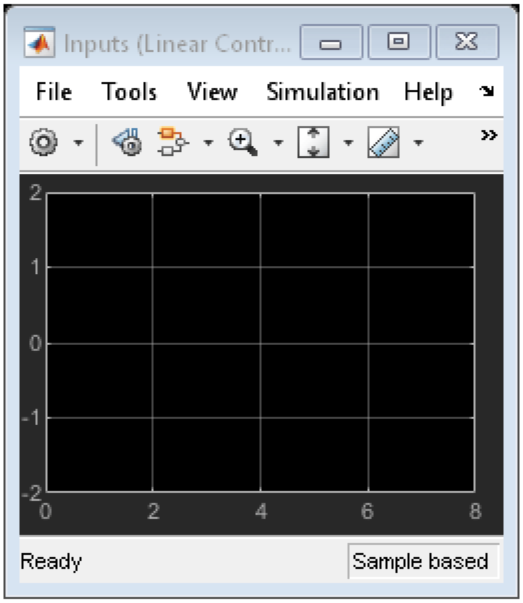
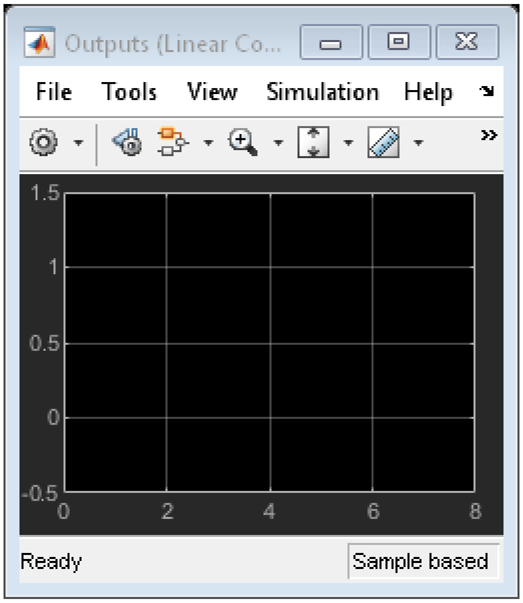
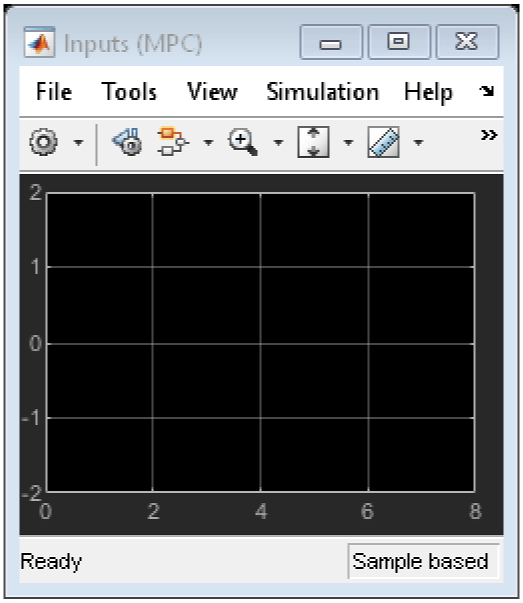
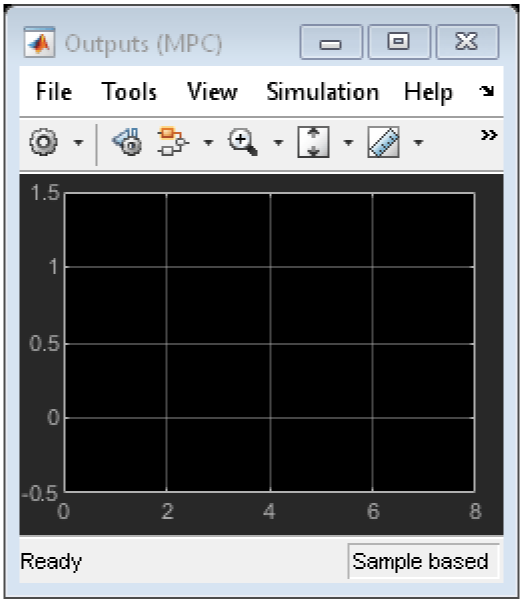

Inputs (Linear Contr...
File
Tools
View
Simulation
Help
2
1
0
-1
-2
0
2
4
6
8
Ready
Sample based

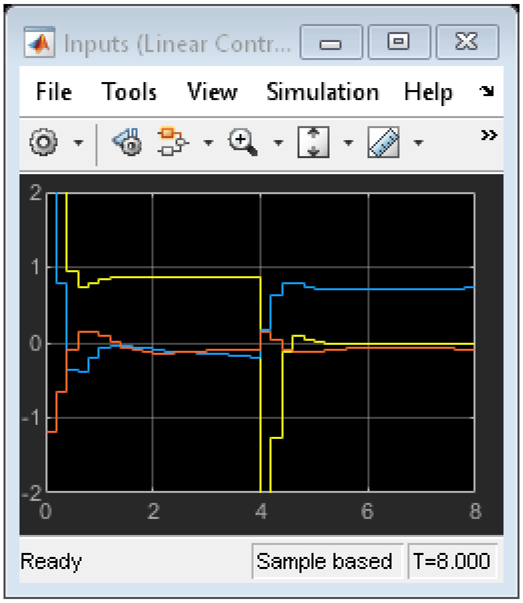

Outputs (Linear Co...
File
Tools
View
Simulation
Help
1.5
1
0.5
0
-0.5
0
2
4
6
8
Ready
Sample based

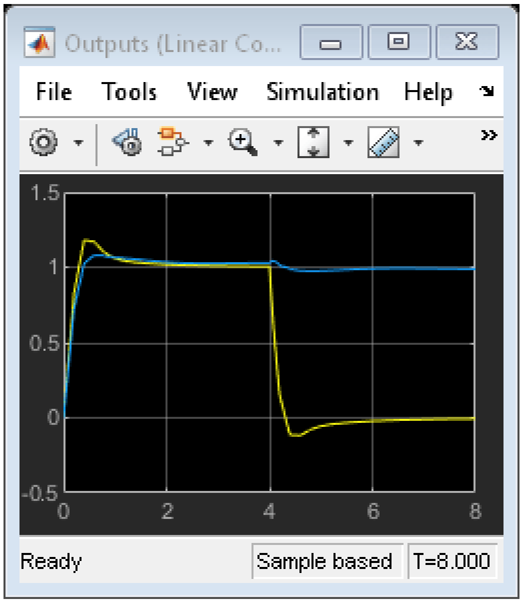


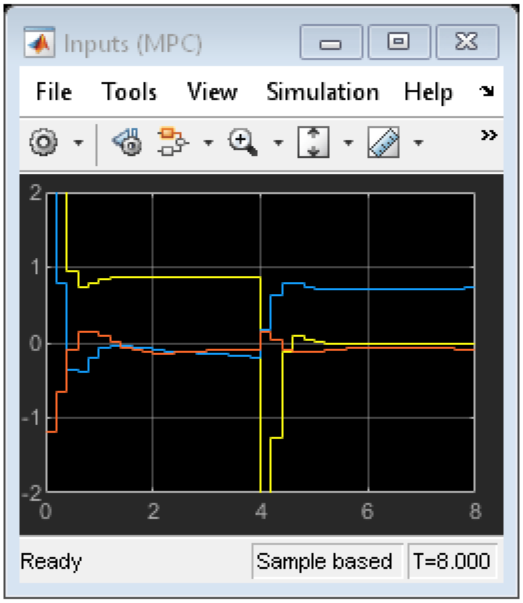


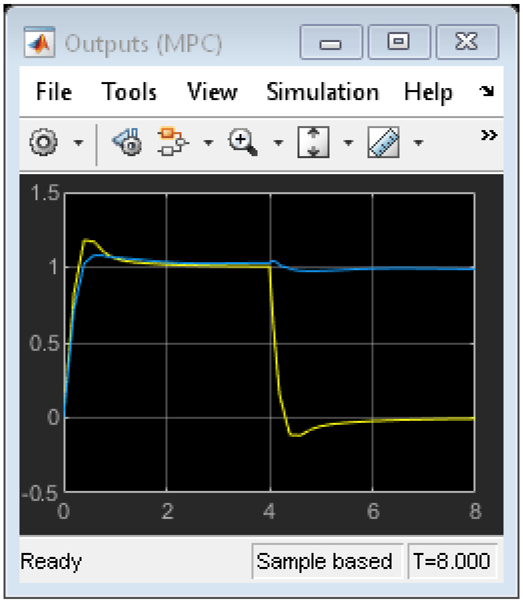
Run the closed loop simulation for 12 seconds.

```
sim(mdl3)
```


Inputs (Linear Contr...
File
Tools
View
Simulation
Help
2
1
0
-1
-2
0
2
4
6
8
Ready
Sample based
T=8.000


Outputs (Linear Co...
File
Tools
View
Simulation
Help
1.5
1
0.5
0
-0.5
0
2
4
6
8
Ready
Sample based
T=8.000





The inputs and outputs signals look identical for both loops. Also note that the manipulated variable are no longer bounded by the previous constaints.

Compare Simulation Results

```
fprintf('Compare output trajectories: ||ympc-ylin|| = %g\n',norm(ympc-ylin));
disp('The MPC controller and the linear controller produce the same closed-loop trajectories.');
```

```
Compare output trajectories: ||ympc-ylin|| = 9.5904e-15
The MPC controller and the linear controller produce the same closed-loop trajectories.
```

As expected, there is only a negligible difference due to numerical errors.

Close all open Simulink models without saving any change.

```
bdclose all
```

See Also

MPC Controller | **MPC Designer** | `mpc`

More About

- “Model Predictive Control of a Single-Input-Single-Output Plant” on page 3-47
- “Model Predictive Control of a Multi-Input Single-Output Plant” on page 3-51